FOCUS Coil-to-Surface Separation Objective Function

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General

FOCUS can optimize for the distance between coils and an arbitrary surface through the coil-to-surface separation objective function, f_{cs} given in Equation 1. This objective function includes a penalty function that can constrain the minimum coil-to-surface separation. This objective function can also maximize the average coil to surface separation. The coils are enforced to be periodic and the i-th coil is parameterized as $\boldsymbol{r_i}(\zeta) \in C^2[0, 2\pi]$. The surface is parameterized with $\theta \in [0, 2\pi]$ and $\varphi \in [0, 2\pi]$. The surface's Jacobian is given as $\sqrt{g}$.

$$f_{cs} = \frac{1}{N_c} \sum_{i=1}^{N_c} \frac{1}{L_i} \int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} \left(H_{-\Delta}(-|\boldsymbol{r_i} - \boldsymbol{r_s}|) \left(\alpha \left(\Delta - |\boldsymbol{r_i} - \boldsymbol{r_s}| \right) \right)^{\beta} + \sigma \left| \boldsymbol{r_i} - \boldsymbol{r_s} \right|^{-\gamma} \right) \sqrt{g} \, d\theta \, d\varphi \left| \boldsymbol{r_i}' \right| d\zeta \tag{1}$$

$$\alpha \geq 0 \qquad \beta \geq 2 \qquad \Delta \geq 0 \qquad \sigma \geq 0 \qquad \gamma \geq 1$$

$$H_{-\Delta}(-|\boldsymbol{r_i} - \boldsymbol{r_s}|) \equiv H(\Delta - |\boldsymbol{r_i} - \boldsymbol{r_s}|) = \begin{cases} 1, & |\boldsymbol{r_i} - \boldsymbol{r_s}| < \Delta \\ \frac{1}{2}, & |\boldsymbol{r_i} - \boldsymbol{r_s}| = \Delta \\ 0, & |\boldsymbol{r_i} - \boldsymbol{r_s}| > \Delta \end{cases} \tag{2}$$

First Derivatives

First derivatives of Equation 1 are given below. See the documentation on the length objective function for the equations L_i and δL_i

$$f_{cs} = \frac{1}{N_c} \sum_{i=1}^{N_c} \frac{1}{L_i} \int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} I(\zeta, \boldsymbol{r_i}, \boldsymbol{r_i}') \, d\theta \, d\varphi \, d\zeta \tag{3}$$

$$I = \left(H_{-\Delta}(-|\boldsymbol{r_i} - \boldsymbol{r_s}|) \left(\alpha \left(\Delta - |\boldsymbol{r_i} - \boldsymbol{r_s}| \right) \right)^{\beta} + \sigma \left| \boldsymbol{r_i} - \boldsymbol{r_s} \right|^{-\gamma} \right) \sqrt{g} \left| \boldsymbol{r_i}' \right| \tag{4}$$

$$\begin{aligned} \delta f_{cs} = \frac{1}{N_c} \sum_{i=1}^{N_c} \frac{1}{L_i} \int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} \delta \boldsymbol{r_i} \cdot \left(\frac{\partial}{\partial \boldsymbol{r_i}} - \frac{d}{d\zeta} \frac{\partial}{\partial \boldsymbol{r_i}'} \right) I \, d\theta \, d\varphi \, d\zeta - \\ \frac{1}{N_c} \sum_{i=1}^{N_c} \frac{\delta L_i}{L_i^2} \int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} I \, d\theta \, d\varphi \, d\zeta \end{aligned} \tag{5}$$

$$\frac{\partial I}{\partial \boldsymbol{r_i}} = \Big(-H_{-\Delta}(-\left|\boldsymbol{r_i} - \boldsymbol{r_s}\right|)\,\alpha\,\beta\,(\alpha\,(\Delta - \left|\boldsymbol{r_i} - \boldsymbol{r_s}\right|))^{\beta-1}\left|\boldsymbol{r_i} - \boldsymbol{r_s}\right|^{-1} - \sigma\gamma\left|\boldsymbol{r_i} - \boldsymbol{r_s}\right|^{-\gamma-2}\Big)(\boldsymbol{r_i} - \boldsymbol{r_s})\sqrt{g}\left|\boldsymbol{r_i}'\right| \tag{6}$$

$$\frac{\partial I}{\partial \boldsymbol{r_i}'} = \Big(H_{-\Delta}(-\left|\boldsymbol{r_i} - \boldsymbol{r_s}\right|)\,(\alpha\,(\Delta - \left|\boldsymbol{r_i} - \boldsymbol{r_s}\right|))^{\beta} + \sigma\left|\boldsymbol{r_i} - \boldsymbol{r_s}\right|^{-\gamma}\Big)\sqrt{g}\left|\boldsymbol{r_i}'\right|^{-1}\boldsymbol{r_i}' \tag{7}$$

$$\begin{aligned}\frac{d}{d\zeta}\frac{\partial I}{\partial \boldsymbol{r_i}'} = {} & \Big(H_{-\Delta}(-\left|\boldsymbol{r_i} - \boldsymbol{r_s}\right|)\,(\alpha\,(\Delta - \left|\boldsymbol{r_i} - \boldsymbol{r_s}\right|))^{\beta} + \sigma\left|\boldsymbol{r_i} - \boldsymbol{r_s}\right|^{-\gamma}\Big)\sqrt{g}\Big(- \left|\boldsymbol{r_i}'\right|^{-3}\boldsymbol{r_i}'\cdot\boldsymbol{r_i}''\boldsymbol{r_i}' + \\ & \left|\boldsymbol{r_i}'\right|^{-1}\boldsymbol{r_i}''\Big) + \Big(- H_{-\Delta}(-\left|\boldsymbol{r_i} - \boldsymbol{r_s}\right|)\,\alpha\,\beta\,(\alpha\,(\Delta - \left|\boldsymbol{r_i} - \boldsymbol{r_s}\right|))^{\beta-1}\left|\boldsymbol{r_i} - \boldsymbol{r_s}\right|^{-1}(\boldsymbol{r_i} - \boldsymbol{r_s})\cdot\boldsymbol{r_i}' - \\ & \sigma\gamma\left|\boldsymbol{r_i} - \boldsymbol{r_s}\right|^{-\gamma-2}(\boldsymbol{r_i} - \boldsymbol{r_s})\cdot\boldsymbol{r_i}'\Big)\sqrt{g}\left|\boldsymbol{r_i}'\right|^{-1}\boldsymbol{r_i}'\end{aligned} \tag{8}$$

Let λ be an arbitrary shaping variable of the j-th coil. First derivatives of f_{cs} with respect to λ are given in the following equation.

$$\begin{aligned}\frac{\partial f_{cs}}{\partial \lambda} = {} & \frac{1}{N_c L_j}\int_0^{2\pi}\int_0^{2\pi}\int_0^{2\pi}\frac{\partial \boldsymbol{r_j}}{\partial \lambda}\cdot\left(\frac{\partial}{\partial \boldsymbol{r_j}} - \frac{d}{d\zeta}\frac{\partial}{\partial \boldsymbol{r_j}'}\right) I\,d\theta\,d\varphi\,d\zeta - \\ & \frac{1}{N_c L_j^2}\frac{\partial L_j}{\partial \lambda}\int_0^{2\pi}\int_0^{2\pi}\int_0^{2\pi} I\,d\theta\,d\varphi\,d\zeta\end{aligned} \tag{9}$$

Notes

First derivatives of this objective function do not include boundary terms from integration by parts since the coils are enforced to be periodic. This objective function can only be used for coil type 1, Fourier coils. As of right now FOCUS can only use the plasma boundary as the surface.

How to Use

The coil-to-surface separation objective function weight is initialized as, "weight_cssep". This variable and all other variables in this section are set in the "*.input" file. No changes to the "*.focus" file are necessary. The value of Δ is set by the variable "mincssep". The values of α, β, σ, and γ are set by the variables "cssep_alpha", "cssep_beta", "cssep_sigma", and "cssep_gamma". The old coil-to-surface separation objective function is still implemented and can be used by inputting the variable "cssep_factor" and setting "case_cssep" to 1. FOCUS uses the new objective function if any other value of "case_cssep" is set.