The constraint on coil length can prevent coils to be too long. There are two different forms of length constraint, exponential and quadratic, controlled by `case_length`. Exponential form is used to shorten coil length as much as possible, while quadratic form is forcing coils to have a length of the *targt_length*.

[called by: solvers.]

General

Without a constraint on length, the coils can become arbitrarily long to lower the ripple, and more "wiggles" can potentially be formed to better match the plasma shape. Besides, the total coil length is directly related to the usage of materials, i.e. cost, which is without any relation to the physical requirements. We include an objective function of in exponential form

$$f_L(\mathbf{X}) = \frac{1}{N_C} \sum_{i=1}^{N_C} \frac{e^{L_i}}{e^{L_{i,o}}}, \tag{1}$$

or in quadratic form

$$f_L(\mathbf{X}) = \frac{1}{N_C} \sum_{i=1}^{N_C} \frac{1}{2} \frac{(L_i - Li,o)^2}{L_{i,o}^2}, \tag{2}$$

where $L_i(\mathbf{X})$ is the length of i-th coil,

$$L_i(\mathbf{X}) = \int_0^{2\pi} |\mathbf{x}_i'| \, dt, \tag{3}$$

and $L_{i,0}$ is a user-specified normalization. The variations in L resulting from a variation $\delta \mathbf{x}_i$ is

$$\delta L(\mathbf{X}) = \int_0^{2\pi} \frac{(\mathbf{x}_i' \cdot \mathbf{x}_i'')\mathbf{x}_i' - (\mathbf{x}_i' \cdot \mathbf{x}_i')\mathbf{x}_i''}{(\mathbf{x}_i' \cdot \mathbf{x}_i')^{3/2}} \cdot \delta \mathbf{x}_i \; dt. \tag{4}$$

First derivatives

From Eq.(4), we can calculated the functional derivatives of coil length with respect to the coil geometries,

$$\frac{\delta L}{\delta x} = \frac{y'y''x' + z'z''x' - y'y'x'' - z'z'x''}{(x'x' + y'y' + z'z')^{3/2}} \; ; \tag{5}$$

$$\frac{\delta L}{\delta y} = \frac{x'x''y' + z'z''y' - x'x'y'' - z'z'y''}{(x'x' + y'y' + z'z')^{3/2}} \; ; \tag{6}$$

$$\frac{\delta L}{\delta z} = \frac{x'x''z' + y'y''z' - x'x'z'' - y'y'z''}{(x'x' + y'y' + z'z')^{3/2}} \; . \tag{7}$$

Then the derivatives of coil length with respect to coil parameters are

$$\frac{\partial L}{\partial X_i} = \int_0^{2\pi} \frac{\delta L}{\delta x} \frac{\partial x}{\partial X_i} + \tag{8}$$

Here, X_i is an arbitrary variable $X_i \in \mathbf{X} = \{I^1, X_{c,0}^1, \dots\}$. So the first derivatives of coil length objective function are

$$\frac{\partial f_L}{\partial X_i} = \frac{1}{N_C} \frac{e^{L_i}}{e^{L_{i,o}}} \frac{\partial L}{\partial X_i} \; ; \tag{9}$$

$$\frac{\partial f_L}{\partial X_i} = \frac{1}{N_C} \frac{(Li - Li,o)}{L_{i,o}^2} \; \frac{\partial L}{\partial X_i} \; . \tag{10}$$