

0.1 overview

1. If `case_coils=1`, then the Fourier series will be used for represent the coils.
2. The basic equations about the Fourier representation is,

$$x = X_{c,0} + \sum_{n=1}^N [X_{c,n} \cos(nt) + X_{s,n} \sin(nt)], \quad (1)$$

$$y = Y_{c,0} + \sum_{n=1}^N [Y_{c,n} \cos(nt) + Y_{s,n} \sin(nt)], \quad (2)$$

$$z = Z_{c,0} + \sum_{n=1}^N [Z_{c,n} \cos(nt) + Z_{s,n} \sin(nt)], \quad (3)$$

0.2 Initalization

There are several ways to initialize the coils data.

1. `case_init = 1` : Toroidally placing `Ncoils` circular coils with a radius of `init_radius` and current of `init_current`. The i th coil is placed at $\zeta = \frac{i-1}{Ncoils} \frac{2\pi}{Nfp}$.
2. `case_init = 0` : Read coils data from `ext.focus` file. The format is as following. **This is the most flexible way, and each coil can be different.**

```
# Total number of coils
      16
#-----1-----
#coil_type symm coil_name
      1 0 Mod_001
#Nseg current Ifree Length Lfree target_length
      128 9.844910899889484E+05 1 5.889288927667147E+00 1 1.000000000000000E+00
#NFcoil
      4
#Fourier harmonics for coils ( xc; xs; yc; ys; zc; zs)
      3.044612087666170E+00 8.531153655332238E-01 4.194525679767678E-02 2.139790853335835E-02
      3.243811555342430E-03
      0.000000000000000E+00 3.542408058492299E-16 -9.108712738922674E-16 1.841880477639364E-16
      -1.172175996642087E-16
      -4.456021385977147E-15 8.545613874434043E-16 -3.133154295448265E-16 1.764367073160815E-16
      -1.187904023667544E-16
      0.000000000000000E+00 -5.425716121023922E-02 -8.986316303345250E-02 -2.946386365076052E-03
      -4.487052148209031E-03
      -4.293247278325474E-17 -1.303273952226587E-15 7.710821807870230E-16 -3.156539892466338E-16
      9.395672288215928E-17
      0.000000000000000E+00 9.997301975562740E-01 2.929938238054118E-02 2.436889176706748E-02
      1.013941937492003E-03
#-----2--permanent magnet-----
#coil_type symm coil_name
      2 0 dipole_01
# Lc ox oy oz Ic I mt mp
      1 0.0 0.0 0.0 1 1.0E6 0.0 0.0
#-----3--background Bt Bz-----
#coil_type symm coil_name
      3 0 bg_BtBz_01
# Ic I Lc Bz (Ic control I; Lc control Bz)
      1 1.0E6 0 0.0
.
.
.
```

3. `case_init = -1` : Get coils data from a standard coils.ext file and then Fourier decomposed (normal Fourier transformation and truncated with NF coil harmonics)

0.3 Discretization

1. Discretizing the coils data involves massive triangular functions in nested loops. As shown in Eq.(??), the outside loop is for different discrete points and for each point, a loop is needed to get the summation of the harmonics.
2. To avoid calling triangular functions every operations, it's a better idea to allocate the public triangular arrays.

$$cmt(iD, iN) = \cos(iN \frac{iD}{D_i} 2\pi); iD = 0, coil(icoil)\%D; iN = 0, coil(icoil)\%N \quad (4)$$

$$smt(iD, iN) = \sin(iN \frac{iD}{D_i} 2\pi); iD = 0, coil(icoil)\%D; iN = 0, coil(icoil)\%N \quad (5)$$

3. Using the concept of vectorization, we can also finish this just through matrix operations. This is in **fouriermatrix**.

```
subroutine fouriermatrix(xc, xs, xx, NF, ND)
  nn(0:NF, 1:1) : matrix for N; iN
  tt(1:1, 0:ND) : matrix for angle; iD/ND*2pi
  nt(0:NF,0:ND) : grid for nt; nt = matmul(nn, tt)
  xc(1:1, 0:NF) : cosin harmonics;
  xs(1:1, 0:NF) : sin harmonics;
  xx(1:1, 0:ND) : returned discrete points;

  xx = xc * cos(nt) + xs * sin(nt)
```

4. Actually, in real tests, the new method is not so fast. And parallelizations are actually slowing the speed, both for the normal and vectorized method.