Optimization and Control

Performance Monitoring Expected Value of Control

Limitation



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Challenges

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Less Need for Control...



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Conflict Monitoring and Modulation of Control



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• Example...

High Conflict




Augment Control



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Lowers Conflict



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 Stem completion (and verbal fluency) Interactive Activation Model (McClelland and Rumelhart, 1981) fMRI: ACC activity (Thompson-Schill et al., 1998)



Conflict Monitoring and Control

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Forced Choice RT Task Sequential adjustment effects

(Laming., 1968)



Eriksen Task Sequential adjustment effects (Gratton et al., 1992)



Stroop Task Frequency effects (Tzelgov, 1992)





Dynamics of Conflict: Error trial



Dynamics of Conflict: Post-processing and Conflict



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- Larger ERN for congruent than incongruent stimuli on error trials (Scheffers & Coles, 2000)



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- Discount reward associated with more effortful tasks...

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Botvinick et al. (2009)



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- "top-down" biasing of cortical pathways require to perform task (e.g., "attentional selection", goal maintenance, motor control)
- systemwide changes in parameters (neuromodulation)
 - changes in learning rate, gating of new control signals into regulatory system
 - noise/gain modulation, explore/exploit tradeoff
 - threshold modulation



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Toward a Formal Account



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 Of the candidate set of control signals (≈ currently executable tasks), one(s) with *highest EVC* (that fall within budget) are *selected for allocation*



Neural Architecture of EVC





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Phenomenological experience...



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- "like you're headed towards a storm that's on the other side, maybe a couple of miles away, and you've got to get across the hill and all of a sudden you're sitting there going how am I going to get over that... you have to keep going forward"





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⇒aggregate behavior is not be reducible (easily, at all?) to a simple expression

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 may (ultimately) be able to write down a set of equations that (nearly) fully describe (almost) the entire system



- but:
 - no single, unified objective function, rather many different ones
 - individual components may respond differently in different environments
 - components may adapt at different rates to changes in the environment

• More easily described as:

- amalgam of controlled + automatic processes ("agents")
- each fully rational with respect to its parameters (e.g., objective function for maximizing reward rate, time scales of adaptation)
- compete with each other and are selected among: (Dulberg et al., 2024)
 - using heuristic approximations to optimal computations
 - based on intrinsic biases, experience and exigencies of environment



 \Rightarrow aggregate behavior is not be reducible (easily, at all?) to a simple expression



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- Cost: *serialization* of processing



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Rational Self-Reconfiguration

• Formal analysis of learning speed vs. processing efficiency



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Bayesian optimal process model

(Sagiv, Musslick & Cohen, 2018)

$$\mathbb{E}_{B}[R|t] = \sum_{i=1}^{\min\{N,K\}} \mathbb{P}(\alpha = i) \sum_{j=0}^{i-1} \mathbb{P}_{B}(\text{success on task } j)(1 - jC)$$
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Deep learning applications

(Ravi, Musslick & Cohen, under review)







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 - > use of EM to "flatten" time... EGO and ISC-CI models!