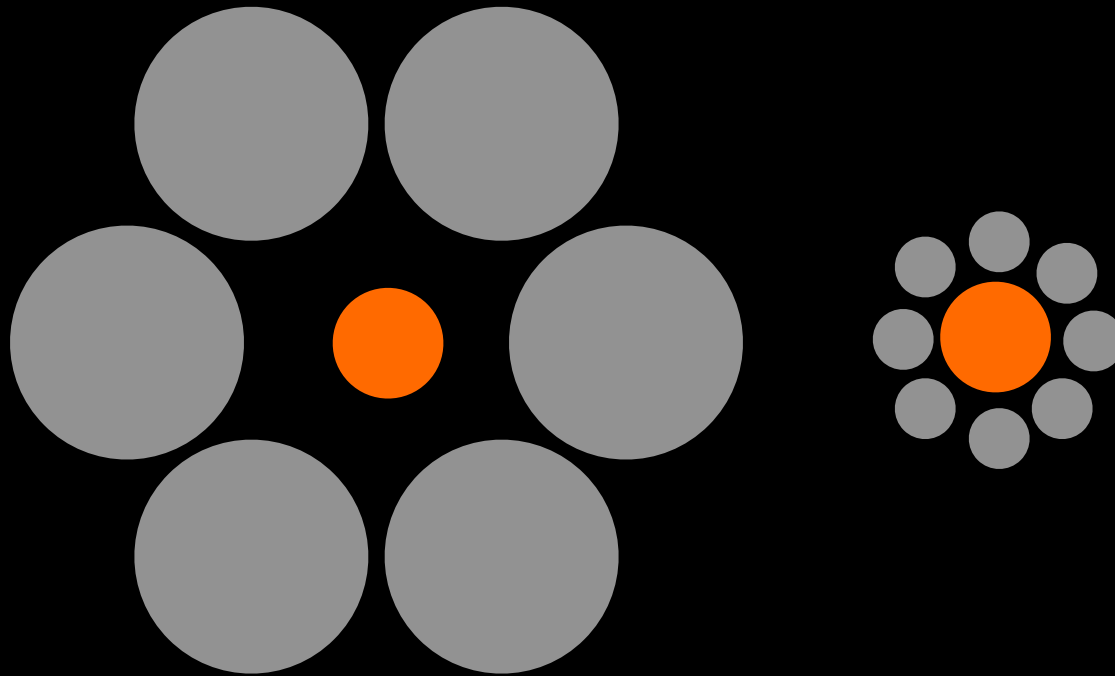


**Constraint Satisfaction,  
Attractor Networks  
and Perception**

# Perceptual Illusions

---

Context influences perception

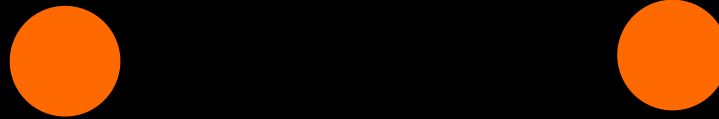


*Ebbinghaus-Titchener Illusion (1901)*

# Perceptual Illusions

---

Context influences perception

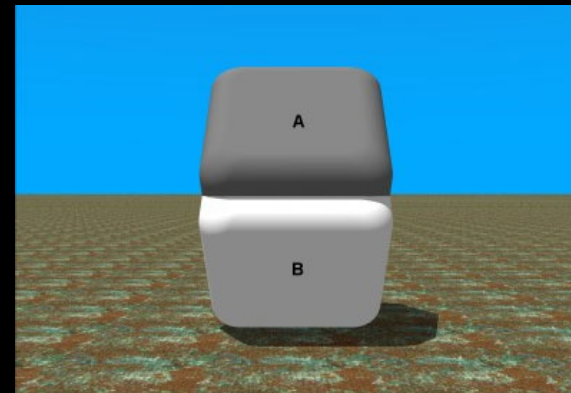
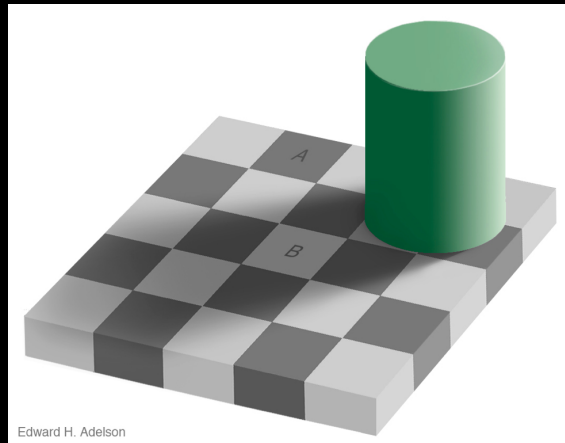


*Ebbinghaus-Titchener Illusion (1901)*

# Perceptual Illusions

---

Context influences perception

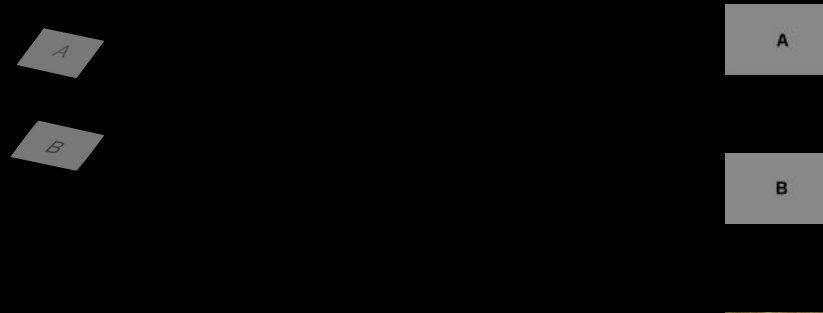


*Edward Adelson (1995)*

# Perceptual Illusions

---

Context influences perception



*Edward Adelson (1995)*

# Top-down Effects

---



*Mask Illusion - Richard Gregory (1970)*

# Top-down Effects

---



*Mask Illusion - Richard Gregory (1970)*

# Top-down Effects

---

Instruction can also be a source of context

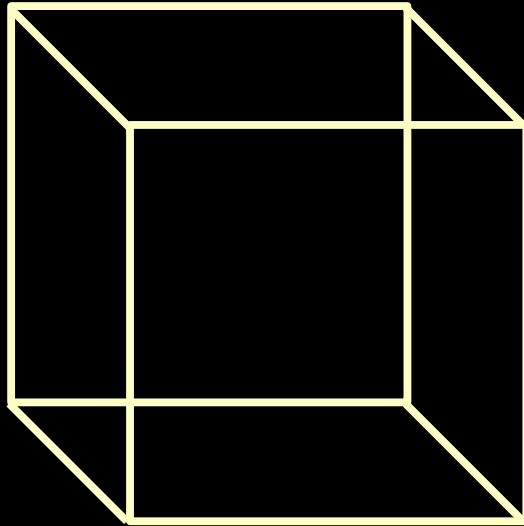




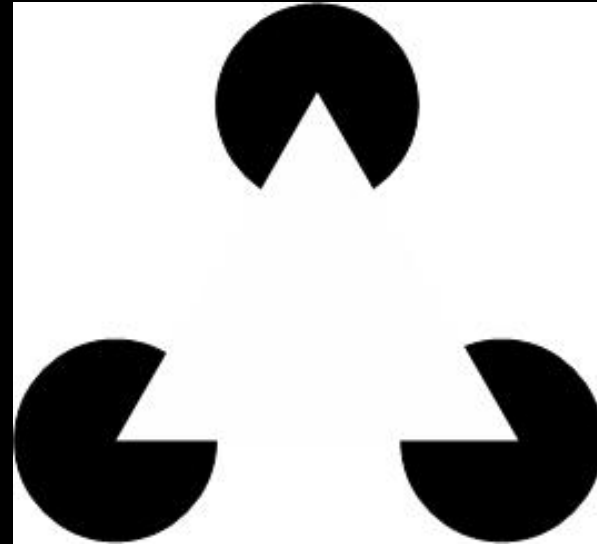
# Gestalt Figures

---

The parts interact to determine the whole



**Necker Cube (1832)**

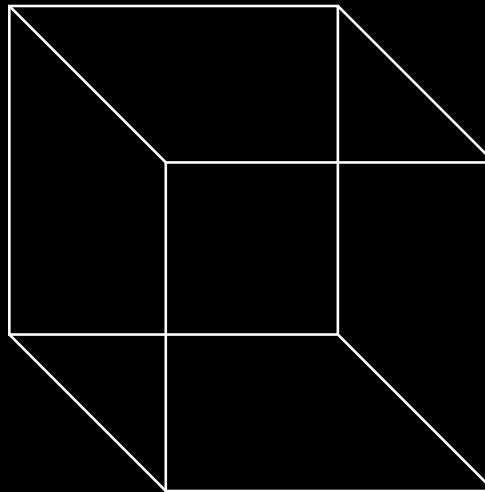


**Kanizsa Triangle (1976)**

# Perceptual Bistability

---

- Instantly perceive a coherent figure (*more or less*)
- Two different interpretations
- Can't perceive both at once
- Alternate between perceptions: *bistability...*

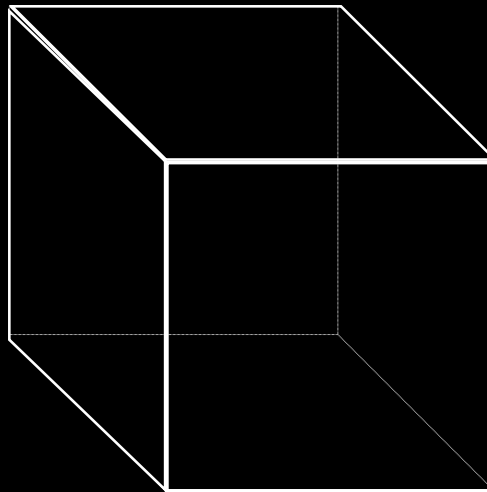


The Necker Cube

# Perceptual Bistability

---

- Instantly perceive coherent figure (more or less)
- Two different interpretations
- Can't perceive both at once
- Alternate between perceptions: *from above*

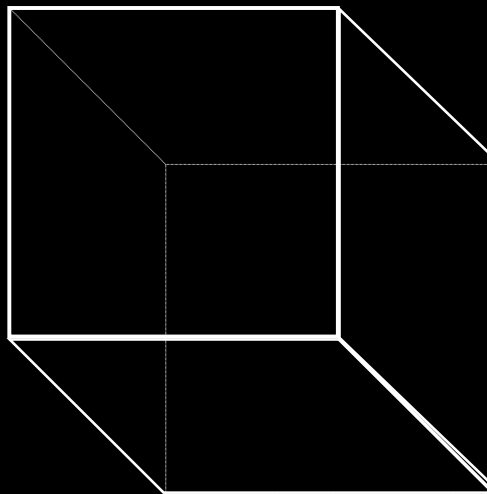


The Necker Cube

# Perceptual Bistability

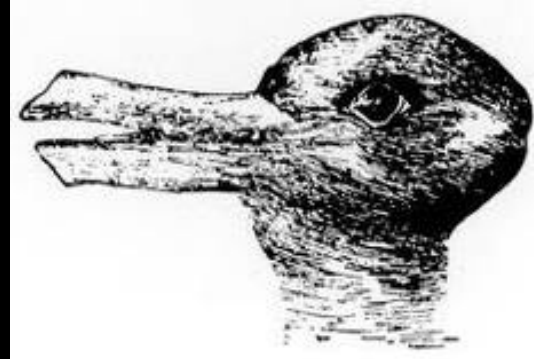
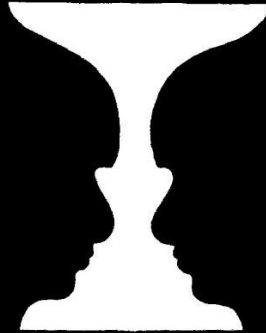
---

- Instantly perceive coherent figure (more or less)
- Two different interpretations
- Can't perceive both at once
- Alternate between perceptions: *from below*



The Necker Cube

# Other Examples



# Gestalt Perception



# Gestalt Perception

---

- Principles that guide perception of the “whole”
  - Similarity
  - Contiguity
  - Continuity
  - Closure
  - Symmetry



# Gestalt Perception

---

- Principles that guide perception of the “whole”
  - Similarity
  - Contiguity
  - Continuity
  - Closure
  - Symmetry
  - **Emergence**





# Gestalt Perception

---

- Principles that guide perception of the “whole”

- Similarity
- Contiguity
- Continuity
- Closure
- Symmetry
- **Emergence**



- Nice heuristic description, but how do these work?

- How do we assess images along all of these dimensions at once, seemingly instantaneously?

*(remember the 100 step rule)*

# **Constraint Satisfaction**

---

# Constraint Satisfaction

---

- These problems can be recast more generally as **constraint satisfaction** problems:

# **Constraint Satisfaction**

---

- **Simultaneously satisfy many interdependent relationships, or “constraints”**  
(e.g., matches between sensory cues, or sensory input and memory representations)

# Constraint Satisfaction

---

- **Simultaneously satisfy many interdependent relationships, or “constraints”**  
(e.g., matches between sensory cues, or sensory input and memory representations)

RED

---

TAE

CAT

# **Constraint Satisfaction**

---

- There may be no perfect solution, so...

# **Constraint Satisfaction**

---

- Look for the best “fit” — one that satisfies as many of the constraints as possible

# **Constraint Satisfaction**

---

- **Some constraints may be more common or important than others**



# **Constraint Satisfaction**

---

- **Connectionist models lend themselves naturally to the solution of such problems...**

# **Constraint Satisfaction**

---

# **Constraint Satisfaction**

---

- Hypotheses = unit activity values

# Constraint Satisfaction

---

- Hypotheses = unit activity values
- **Constraints = connections between units**

# Constraint Satisfaction

---

- Hypotheses = unit activity values
- Constraints = connections between units
- **Importance of constraint = weight of connection**

# Constraint Satisfaction

---

- Hypotheses = unit activity values
- Constraints = connections between units
- Importance of constraint = weight of connection
- *A priori* probability of truth of a hypothesis = biases

# Constraint Satisfaction

---

- Hypotheses = unit activity values
- Constraints = connections between units
- Importance of constraint = weight of connection
- *A priori* probability of truth of a hypothesis = biases
- Evidence for a given hypothesis = external input

# Constraint Satisfaction

---

- Hypotheses = unit activity values
- Constraints = connections between units
- Importance of constraint = weight of connection
- *A priori* probability of truth of a hypothesis = biases
- Evidence for a given hypothesis = external input
- **Satisficing = settling process**



# Constraint Satisfaction

---

- Hypotheses = unit activity values
- Constraints = connections between units
- Importance of constraint = weight of connection
- *A priori* probability of truth of a hypothesis = biases
- Evidence for a given hypothesis = external input
- Satisficing = settling process
- **Success = goodness of fit**

# Constraint Satisfaction

---

- Hypotheses = unit activity values
- Constraints = connections between units
- Importance of constraint = weight of connection
- *A priori* probability of truth of a hypothesis = biases
- Evidence for a given hypothesis = external input
- Satisficing = settling process
- Success = goodness of fit



$$P(\text{Hypothesis}|\text{Data}) = \frac{P(\text{Data}|\text{Hypothesis}) \cdot P(\text{Hypothesis})}{P(\text{Hypothesis}, \text{Data})}$$

# Constraint Satisfaction

- Hypotheses = unit activity values
- Constraints = connections between units
- Importance of constraint = weight of connection
- *A priori* probability of truth of a hypothesis = biases
- Evidence for a given hypothesis = external input
- Satisficing = settling process
- Success = goodness of fit

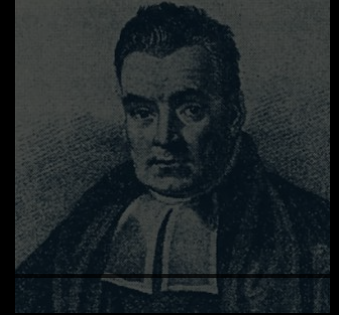


*posterior* ← *likelihood* ← *prior*

$$P(\text{Hypothesis}|\text{Data}) = \frac{P(\text{Data}|\text{Hypothesis}) \cdot P(\text{Hypothesis})}{P(\text{Hypothesis}, \text{Data})}$$

# Constraint Satisfaction

- Hypotheses = unit activity values
- Constraints = connections between units
- Importance of constraint = weight of connection
- *A priori* probability of truth of a hypothesis = biases
- Evidence for a given hypothesis = external input
- Satisficing = settling process
- Success = goodness of fit



*posterior*

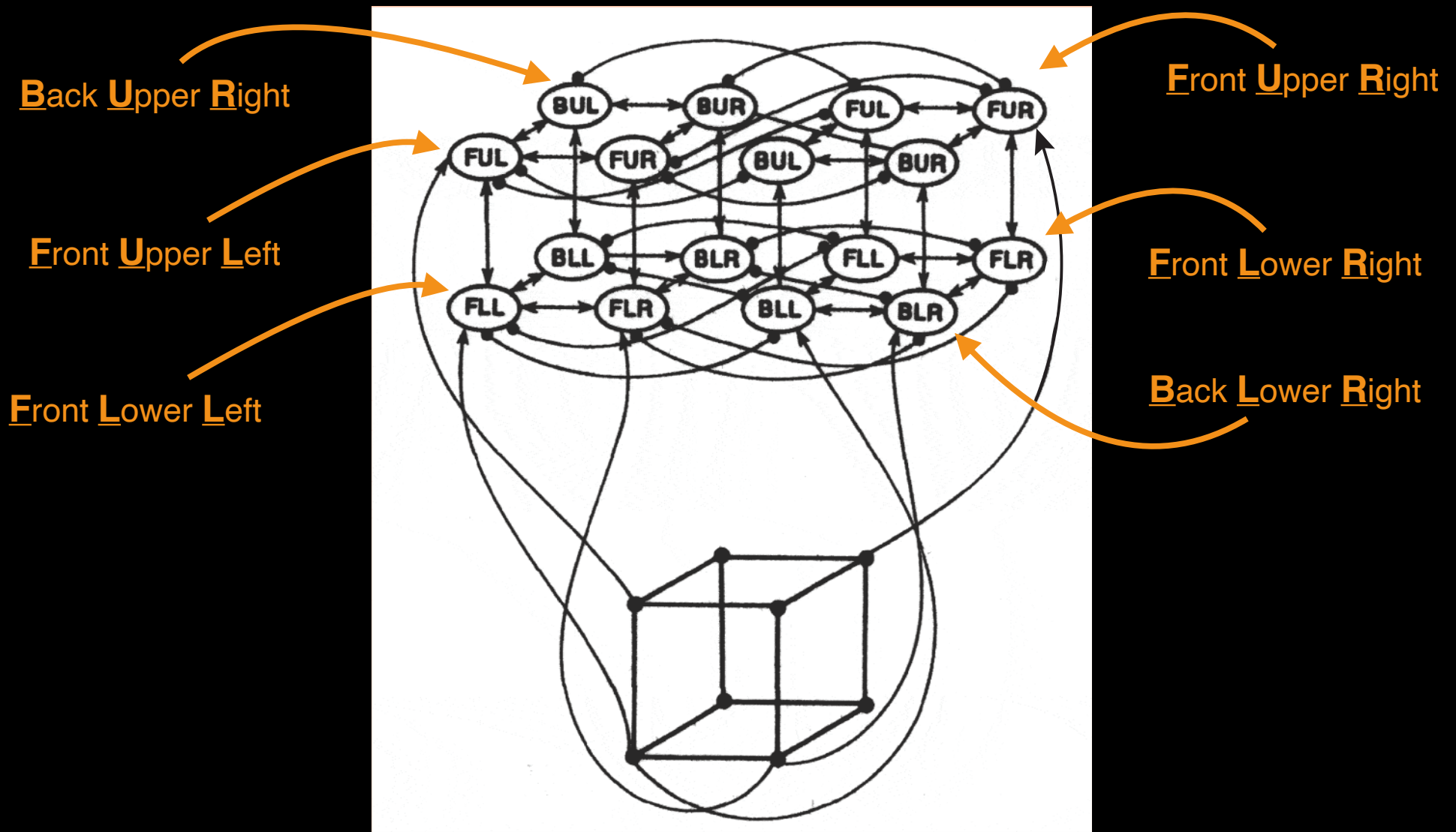
*likelihood*

*prior*

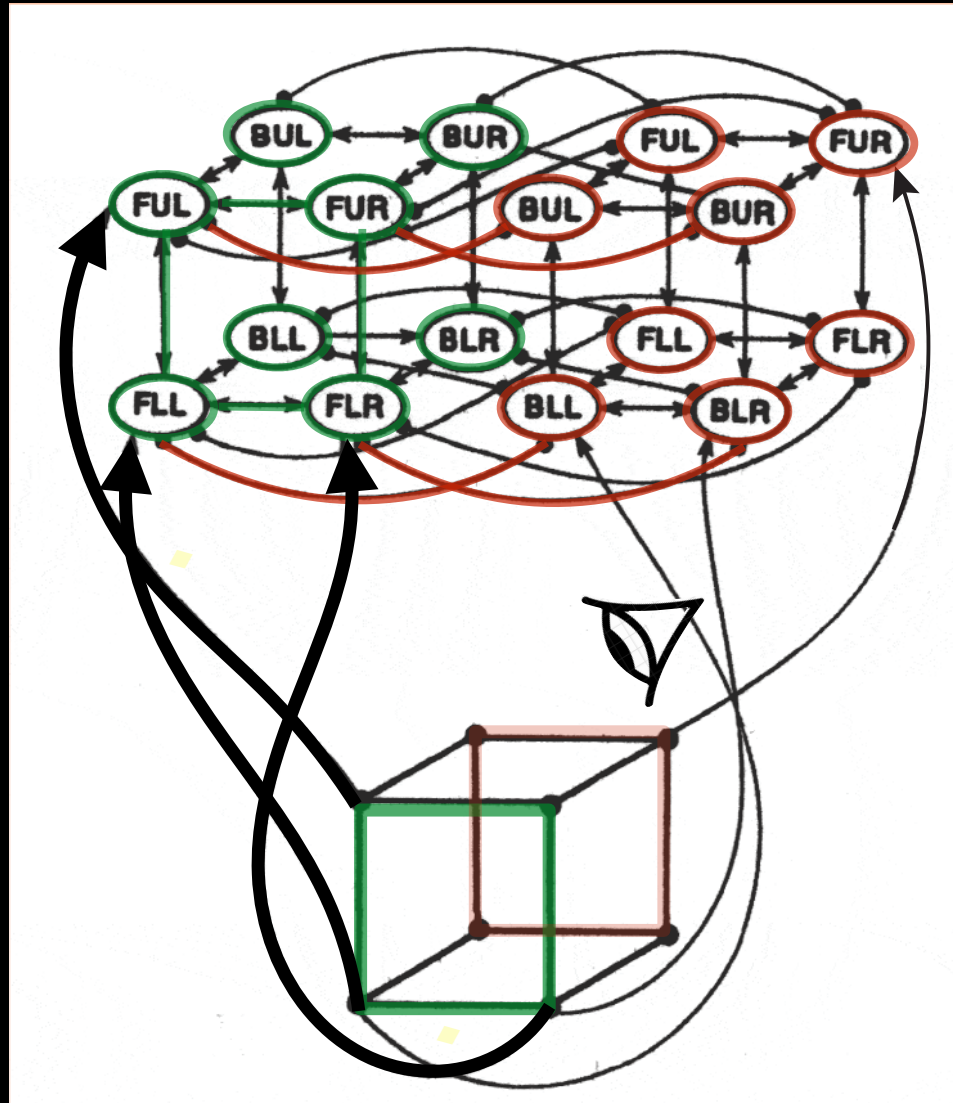
$$P(\text{Hypothesis}|\text{Data}) = \frac{P(\text{Data}|\text{Hypothesis}) \cdot P(\text{Hypothesis})}{P(\text{Hypothesis}, \text{Data})}$$

- How can this be applied to psychological phenomena?

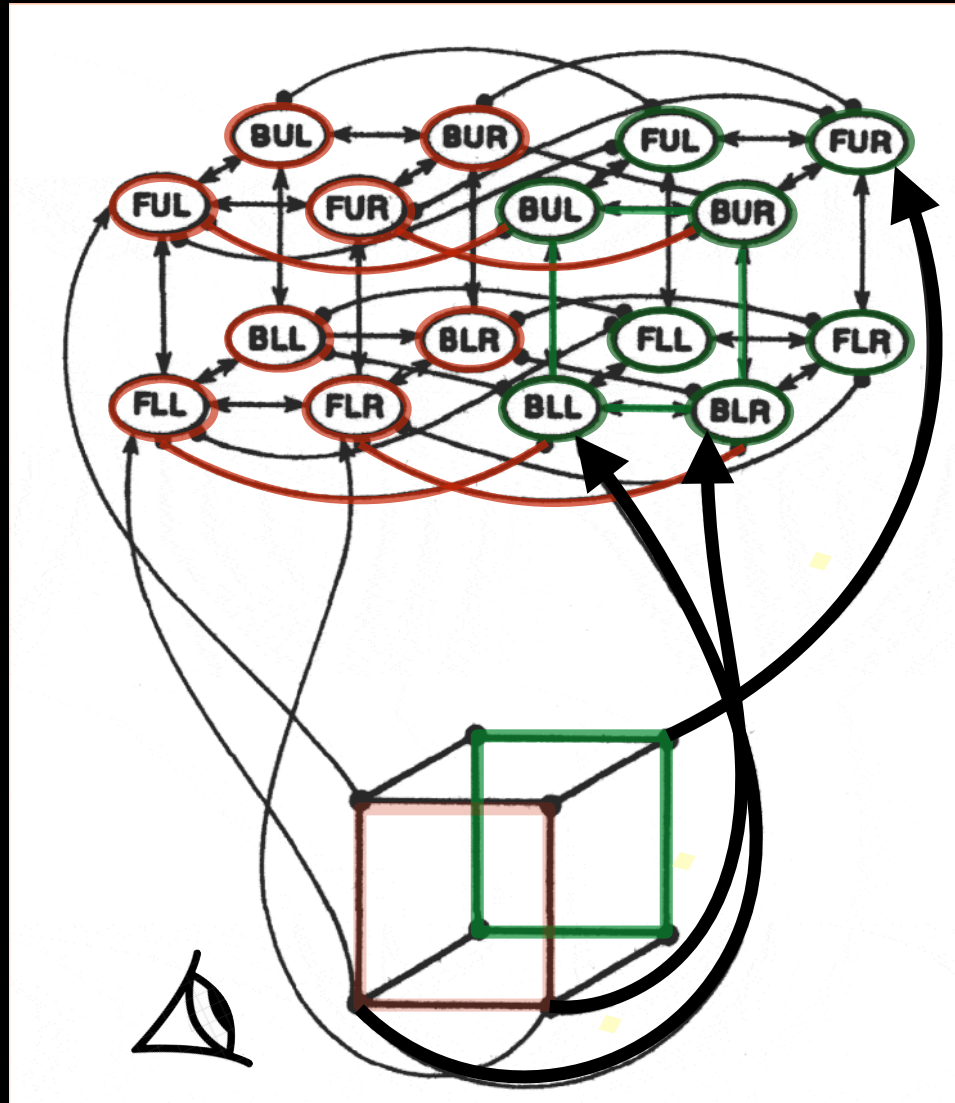
# Constraint Satisfaction Network



# Constraint Satisfaction Network

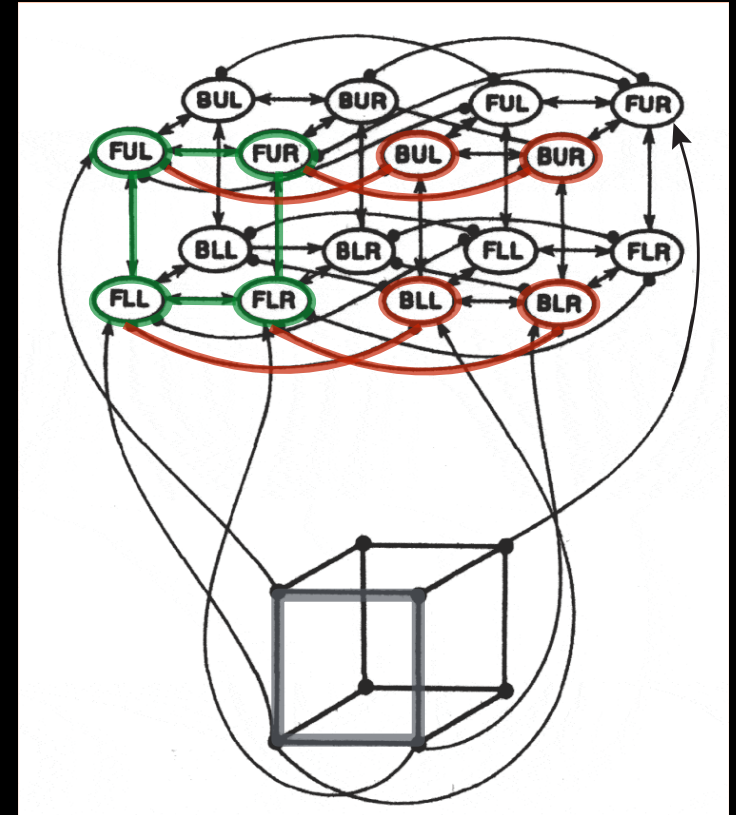


# Constraint Satisfaction Network



# Constraint Satisfaction Network

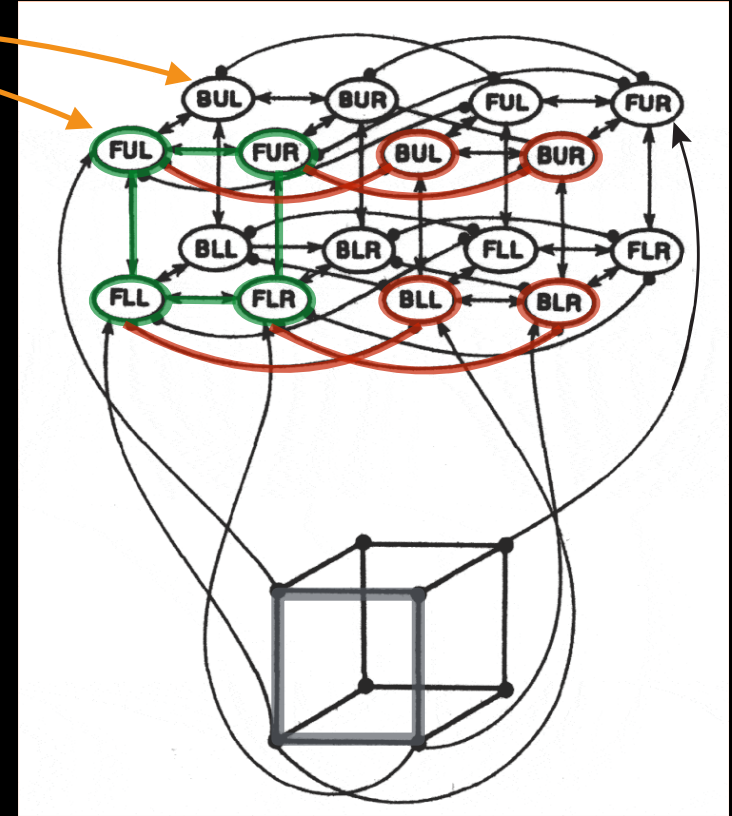
---





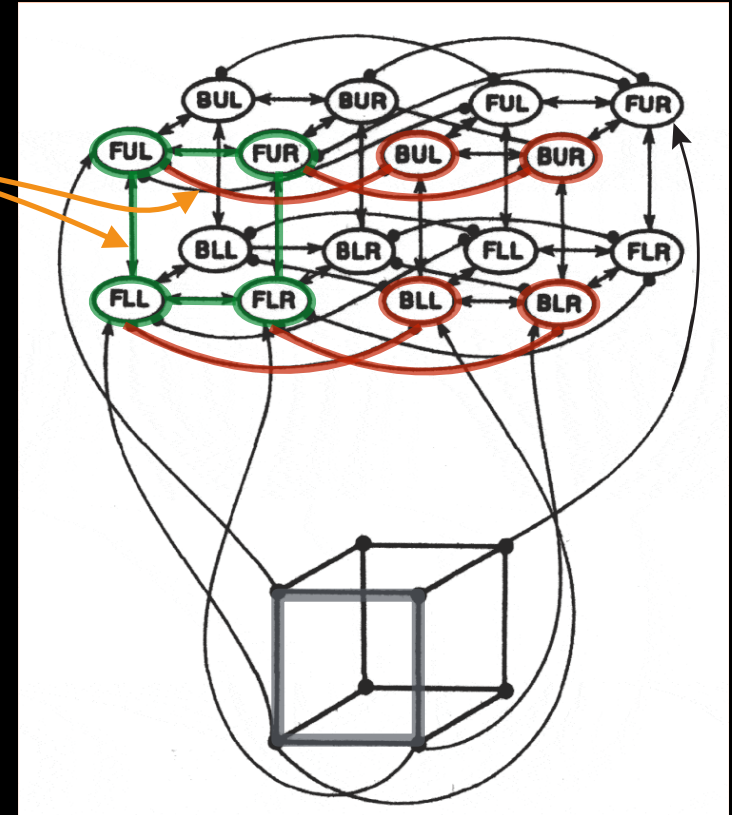
# Constraint Satisfaction Network

- *Hypotheses* = unit activity values



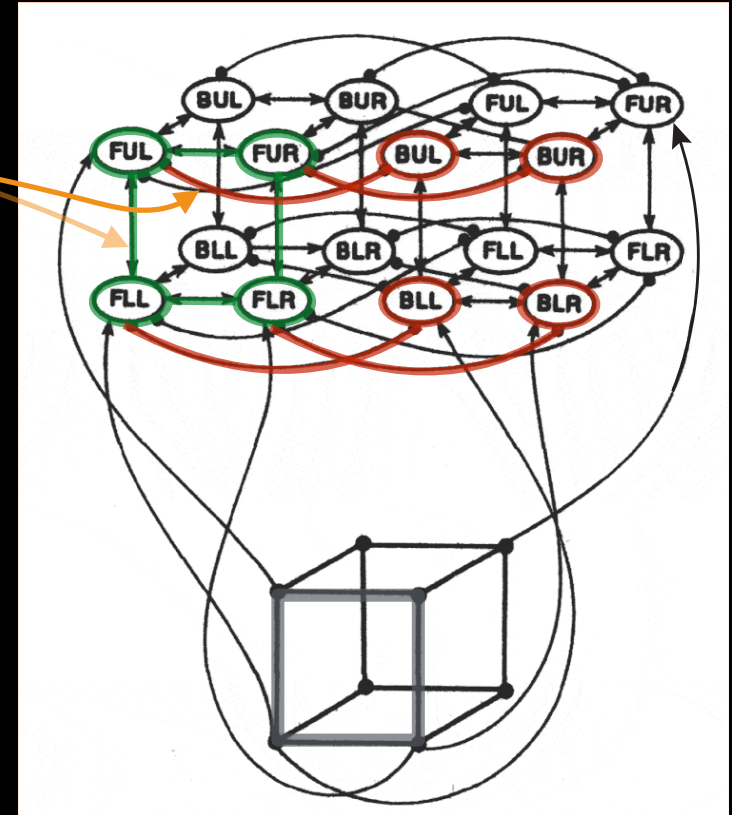
# Constraint Satisfaction Network

- **Constraints** = connections between units



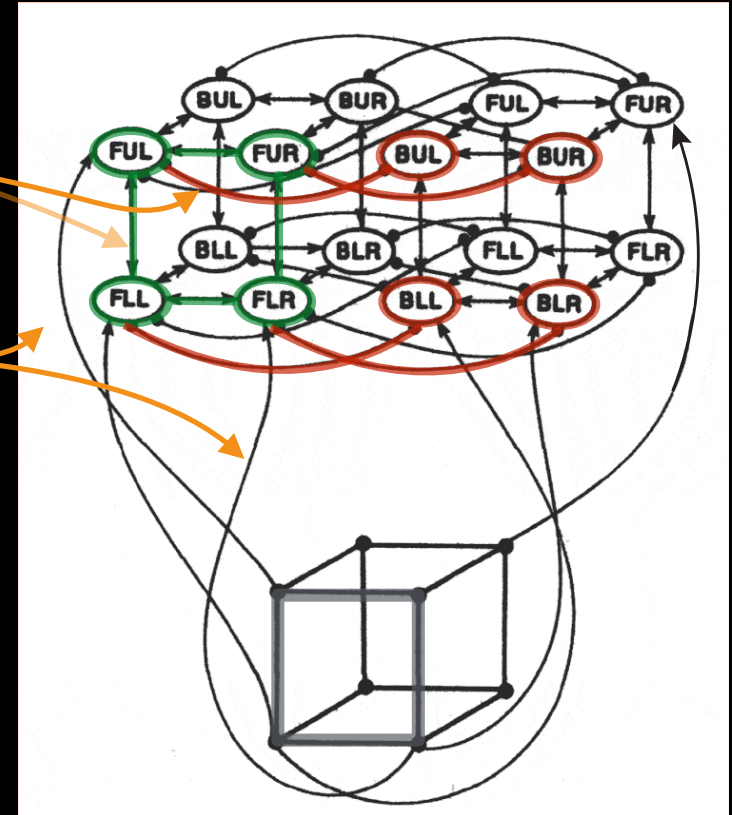
# Constraint Satisfaction Network

- *Importance of constraint* = weight of connection



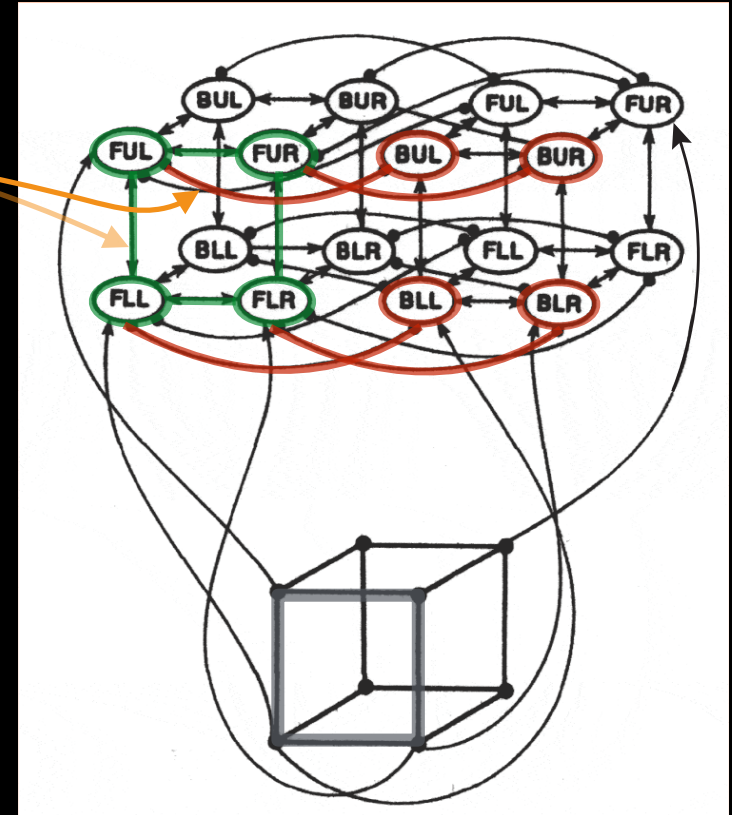
# Constraint Satisfaction Network

- **Evidence** (for a given hypothesis) = external input



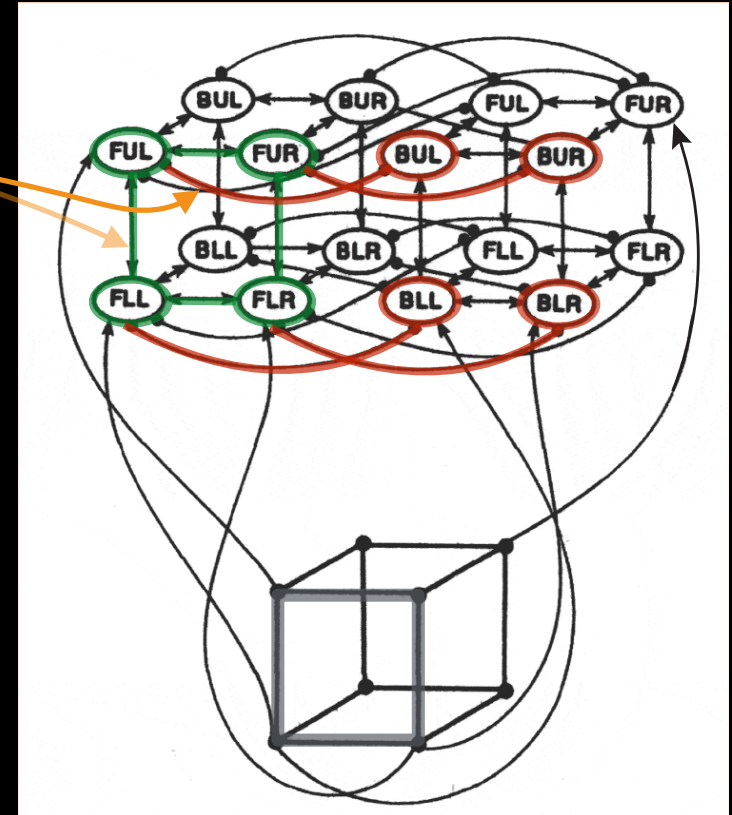
# Constraint Satisfaction Network

- *A priori probability* (for a given hypothesis) = biases



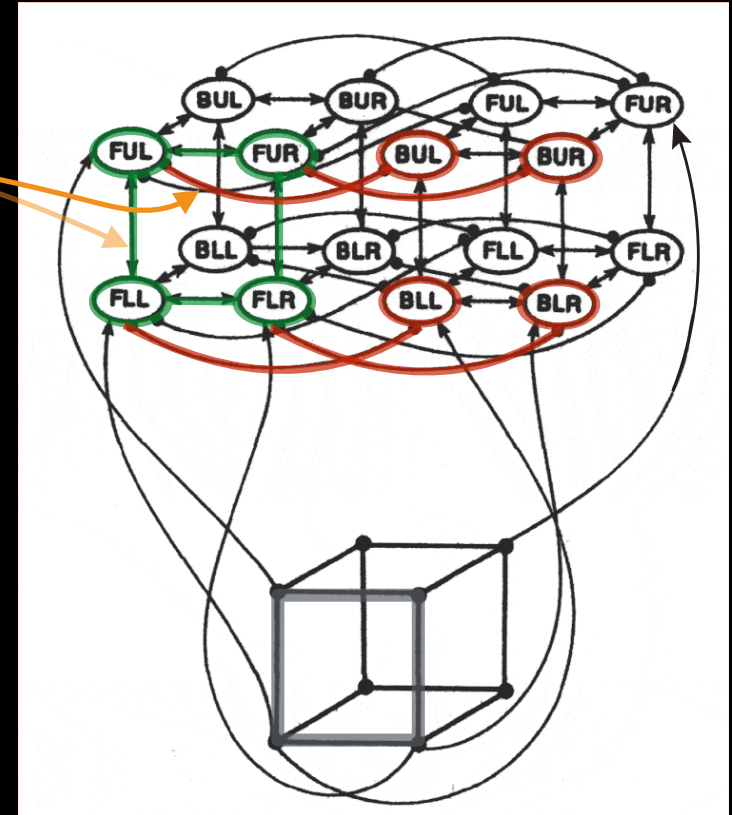
# Constraint Satisfaction Network

- *Inference* = settling process





# Constraint Satisfaction Network



- **Success** = goodness of fit

- How can we formalize this?



# Hopfield Networks, Energy & "State Space" Dynamics

# Hopfield Networks, Energy & "State Space" Dynamics

- **State: vector of unit activation values**  
(*state space = range of all possible vector values*)



# Hopfield Networks, Energy & "State Space" Dynamics

[ FUL FLL BUL BLL FUR ... ]

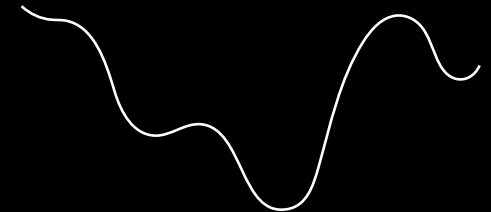
- Energy of each state:  $E = -\frac{\sum_{ij} w_{ij} a_i a_j}{2}$   
(opposite of Goodness)

# Hopfield Networks, Energy & "State Space" Dynamics

[ FUL FLL BUL BLL FUR ... ]

$$E = - \frac{\sum_{ij} w_{ij} a_i a_j}{2}$$

- Energy surface: plot of energy for every state

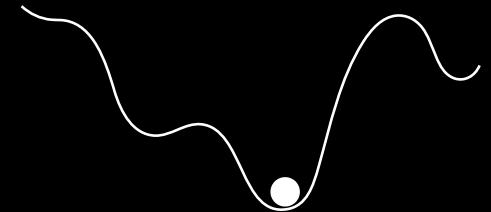


# Hopfield Networks, Energy & "State Space" Dynamics

[ FUL FLL BUL BLL FUR ... ]

$$E = - \frac{\sum_{ij} w_{ij} a_i a_j}{2}$$

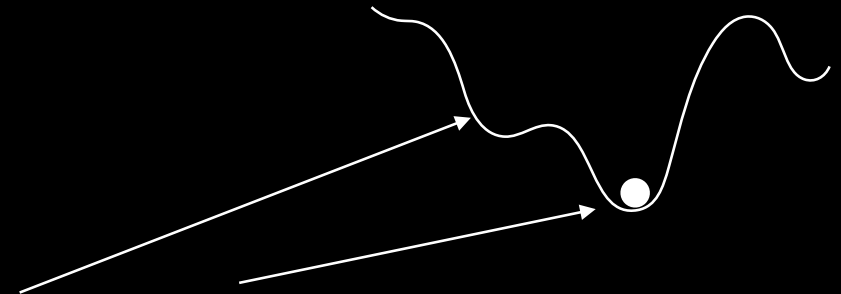
- Dynamics: traversal of energy surface



# Hopfield Networks, Energy & "State Space" Dynamics

[ FUL FLL BUL BLL FUR ... ]

$$E = - \frac{\sum_{ij} w_{ij} a_i a_j}{2}$$

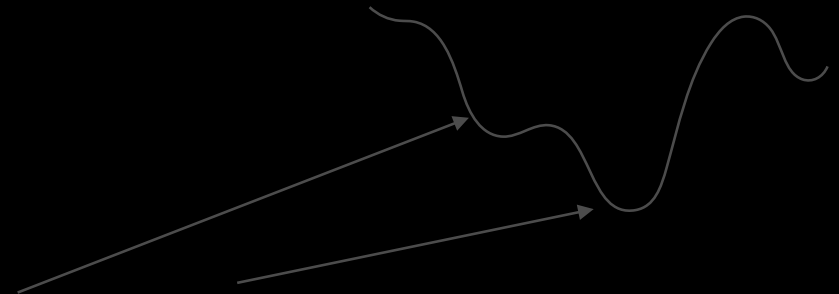


- Minima: points of lowest energy (local & global)

# Hopfield Networks, Energy & "State Space" Dynamics

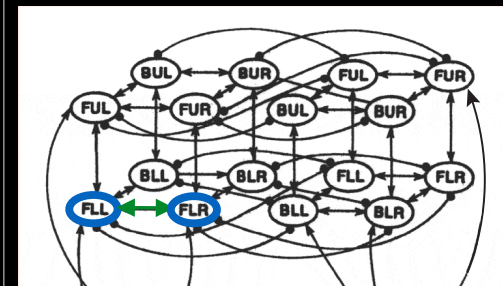
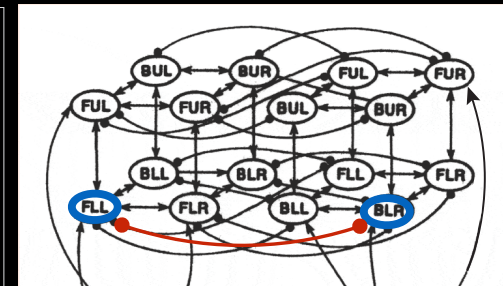
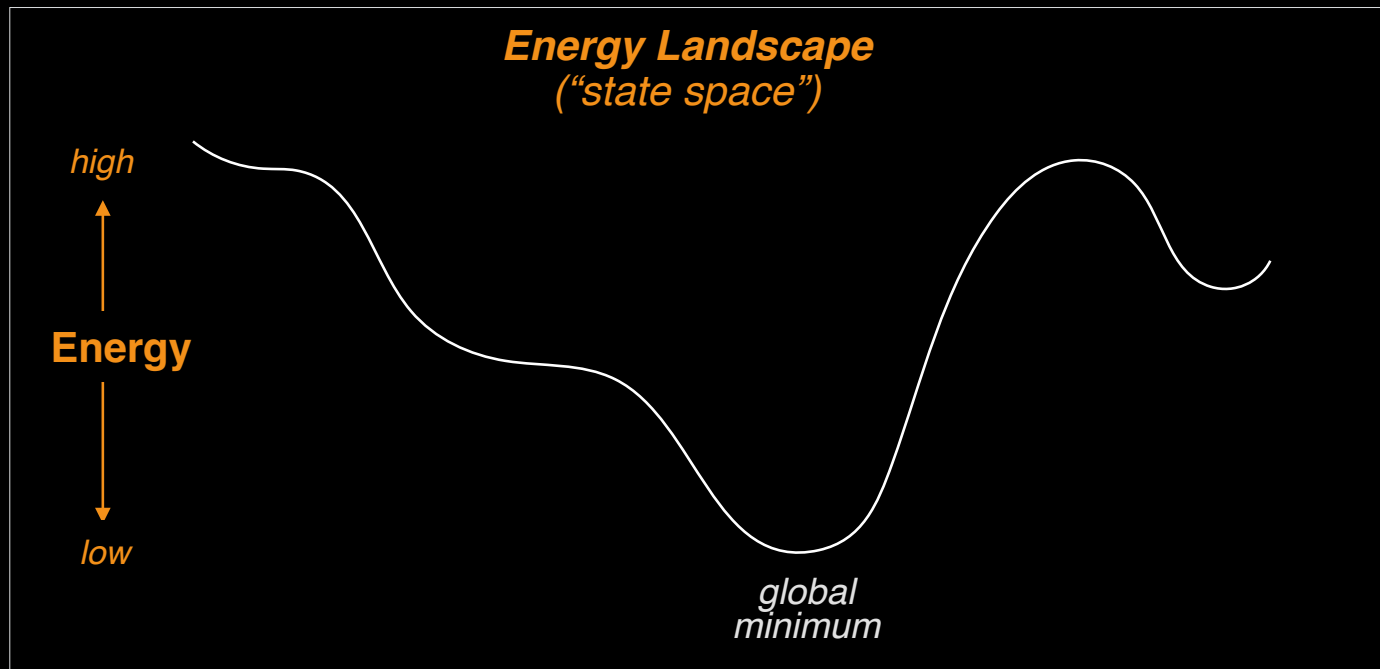
[ FUL FLL BUL BLL FUR ... ]

$$E = - \frac{\sum_{ij} w_{ij} a_i a_j}{2}$$



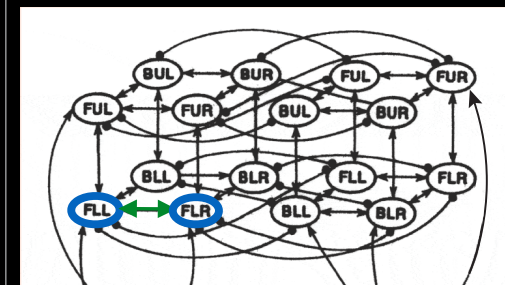
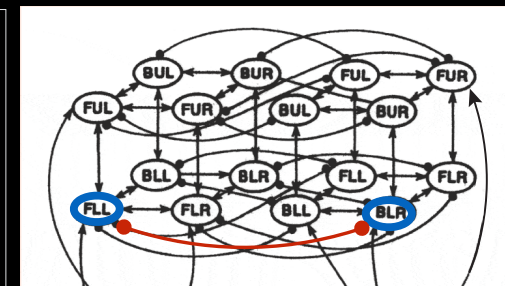
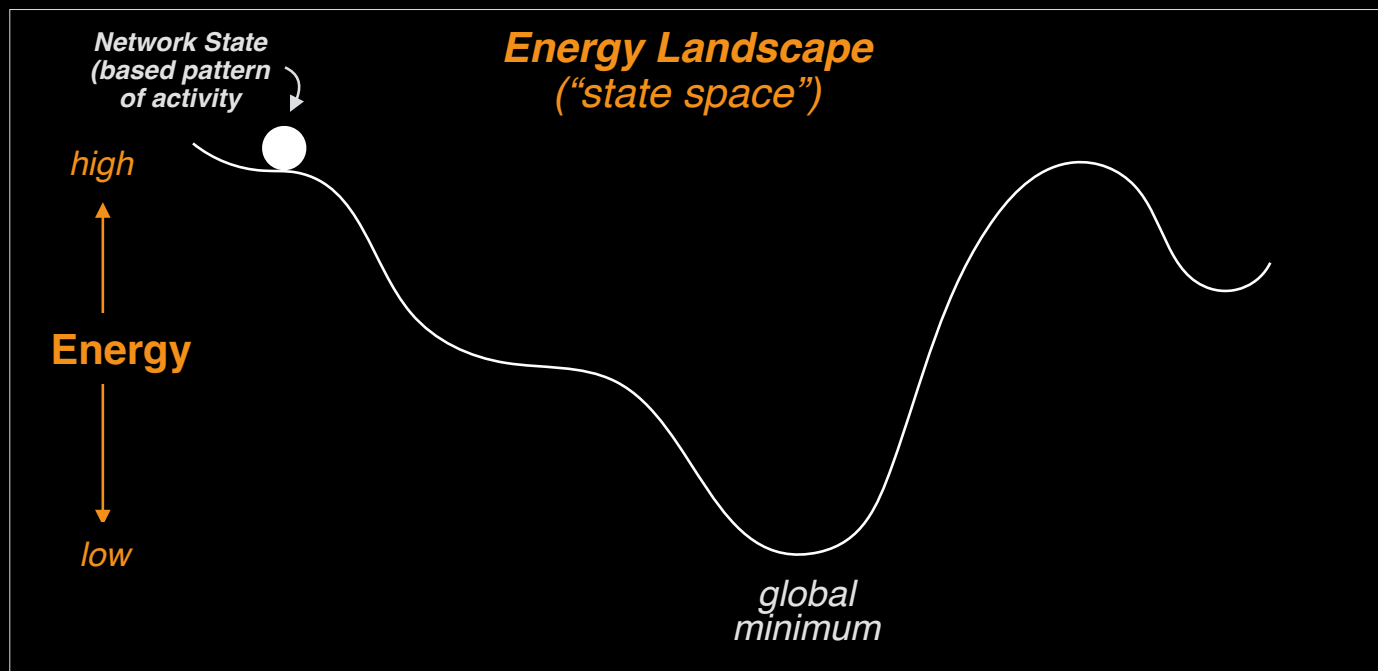
- Under proper assumptions, can prove that system will flow down hill

# Energy Landscape

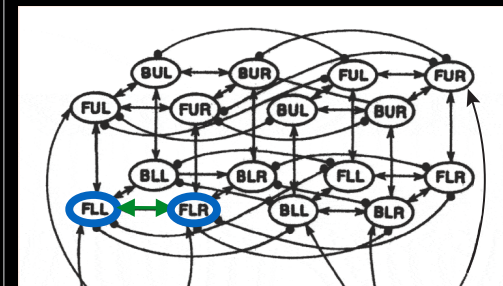
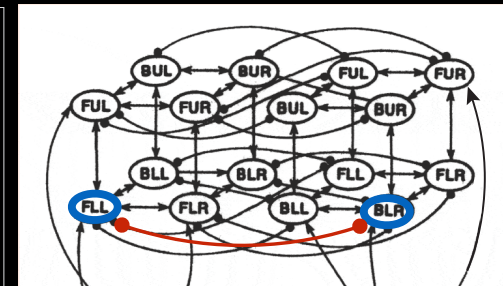
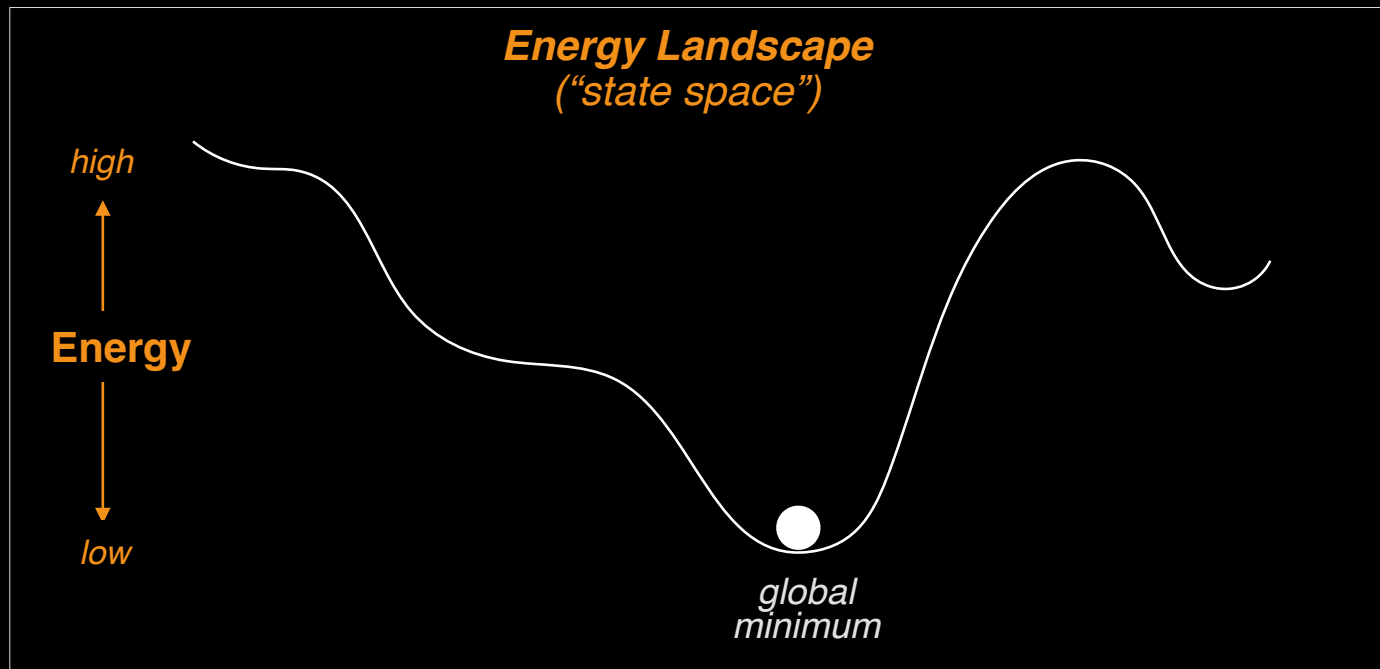




# Energy Landscape



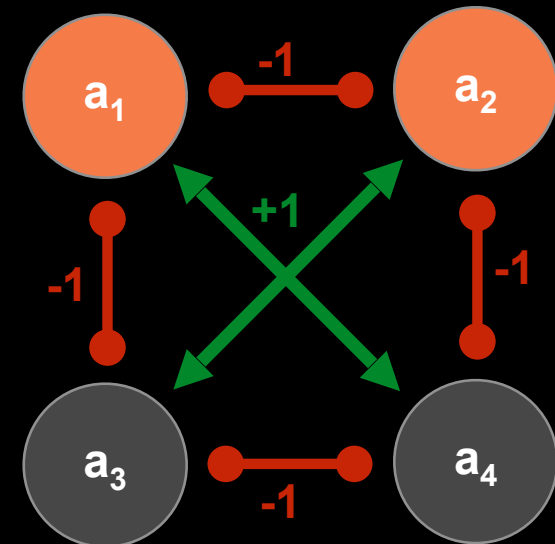
# Energy Landscape



# An Example

$$E = - \frac{\sum_{ij} w_{ij} a_i a_j}{2}$$

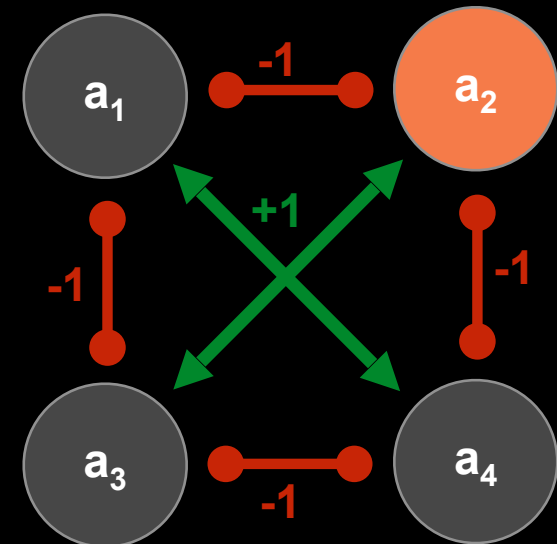
| Update        | $a_1$ | $a_2$ | $a_3$ | $a_4$ | Energy |
|---------------|-------|-------|-------|-------|--------|
| Initial State | 1     | 1     | 0     | 0     | +0.5   |
|               |       |       |       |       |        |



# An Example

$$E = - \frac{\sum_{ij} w_{ij} a_i a_j}{2}$$

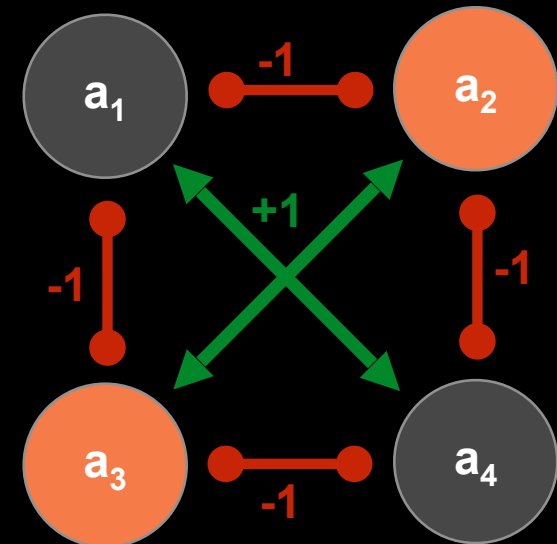
| Update | $a_1$ | $a_2$ | $a_3$ | $a_4$ | Energy |
|--------|-------|-------|-------|-------|--------|
| $a_1$  | 0     | 1     | 0     | 0     | 0      |



# An Example

$$E = -\frac{\sum_{ij} w_{ij} a_i a_j}{2}$$

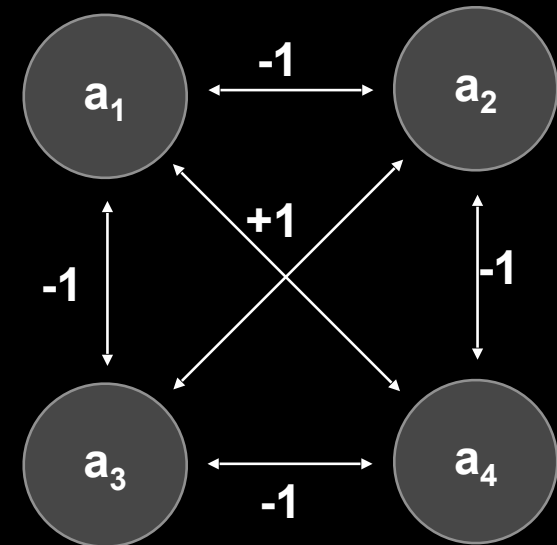
| Update | $a_1$ | $a_2$ | $a_3$ | $a_4$ | Energy |
|--------|-------|-------|-------|-------|--------|
|        | 1     | 1     | 0     | 0     | +0.5   |
| $a_1$  | 0     | 1     | 0     | 0     | 0      |
| $a_3$  | 0     | 1     | 1     | 0     | -0.5   |



# An Example

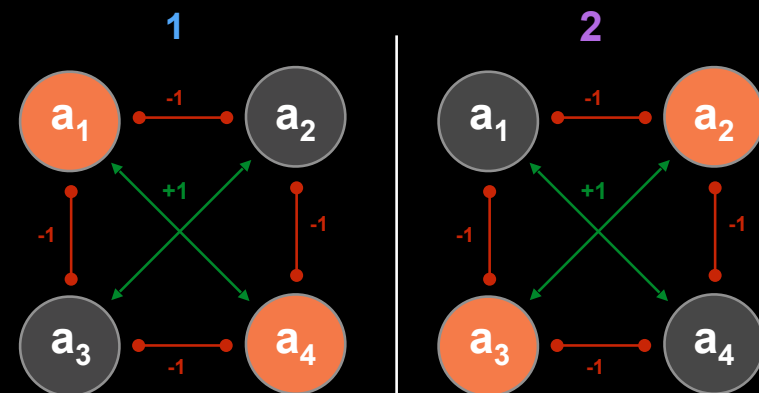
$$E = -\frac{\sum_{ij} w_{ij} a_i a_j}{2}$$

| Update        | $a_1$ | $a_2$ | $a_3$ | $a_4$ | Energy |
|---------------|-------|-------|-------|-------|--------|
| Initial state | 1     | 1     | 0     | 0     | +0.5   |
| $a_1$         | 0     | 1     | 0     | 0     | 0      |
| $a_2$         | 0     | 1     | 1     | 0     | -0.5   |



Two stable states — percepts (or memories)

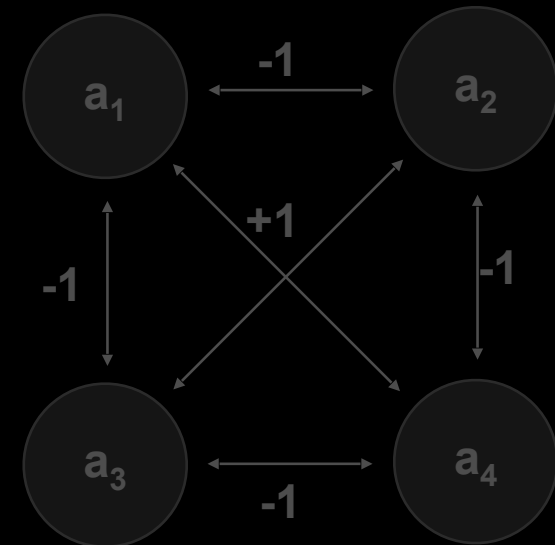
| State | $a_1$ | $a_2$ | $a_3$ | $a_4$ | Energy |
|-------|-------|-------|-------|-------|--------|
| 1     | 1     | 0     | 0     | 1     | -0.5   |
| 2     | 0     | 1     | 1     | 0     | -0.5   |



# An Example

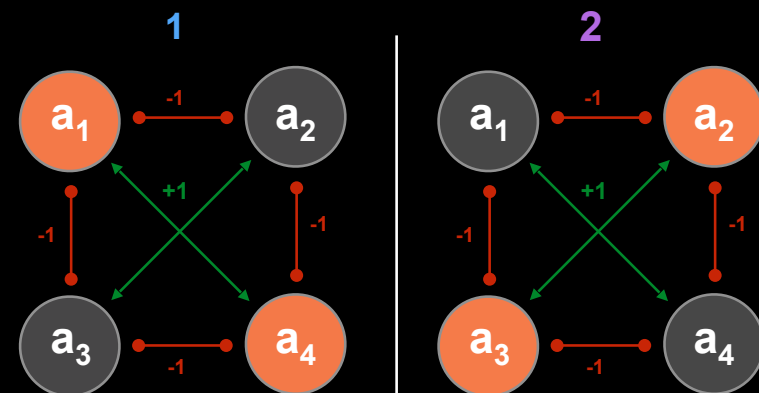
$$E = - \frac{\sum_{ij} w_{ij} a_i a_j}{2}$$

| Update        | $a_1$ | $a_2$ | $a_3$ | $a_4$ | Energy |
|---------------|-------|-------|-------|-------|--------|
| Initial state | 1     | 1     | 0     | 0     | +0.5   |
| $a_1$         | 0     | 1     | 0     | 0     | 0      |
| $a_2$         | 0     | 1     | 1     | 0     | -0.5   |



Two stable states — percepts (or memories)

| State | $a_1$ | $a_2$ | $a_3$ | $a_4$ | Energy |
|-------|-------|-------|-------|-------|--------|
| 1     | 1     | 0     | 0     | 1     | -0.5   |
| 2     | 0     | 1     | 1     | 0     | -0.5   |



# **Energy Landscapes**



# **Energy Landscapes**

---

- **A surface that is the energy of the network as a function of the activity of its units**

# **Energy Landscapes**

---

- **The state of the system is a point on this surface**

# **Energy Landscapes**

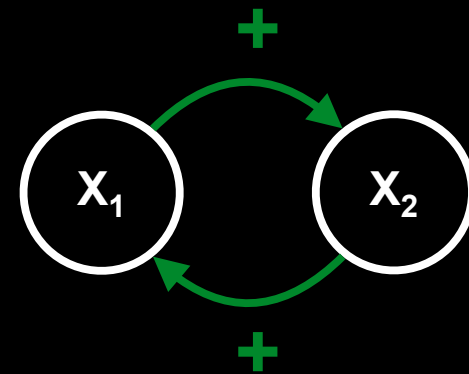
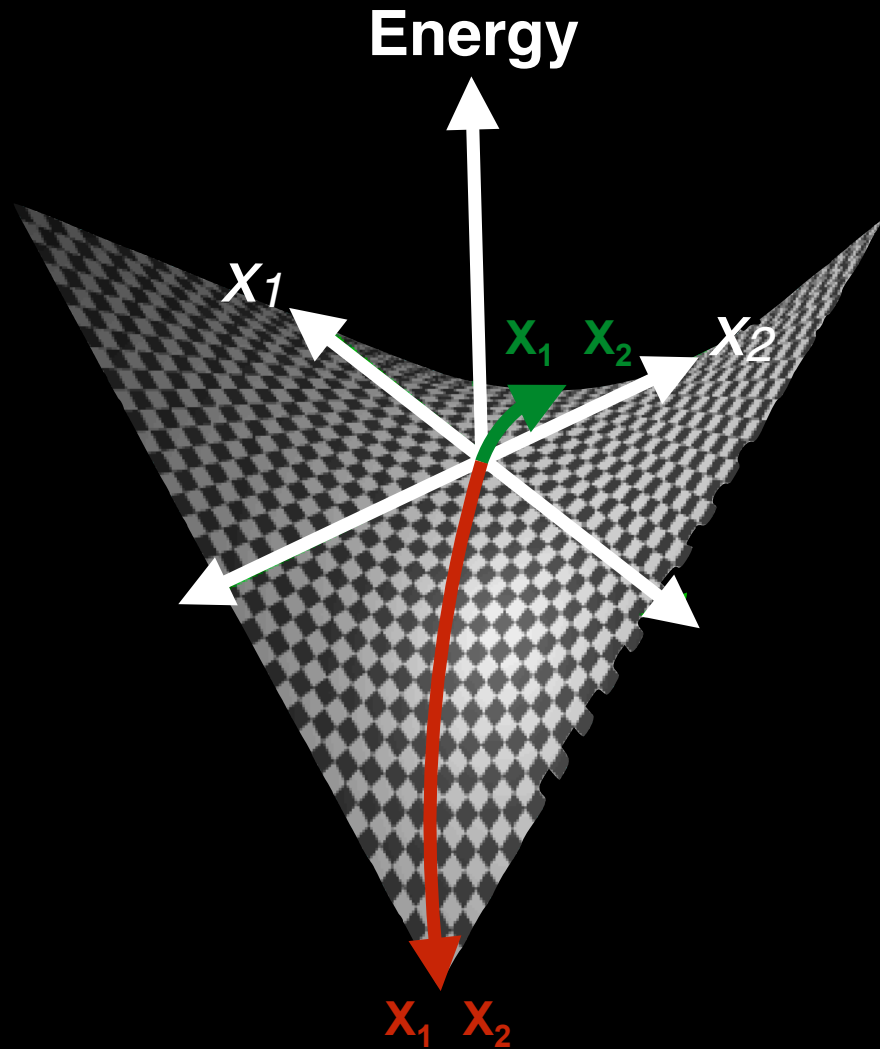
---

- **The settling process of the network is the downhill trajectory of this network along this surface**

# Energy Landscapes

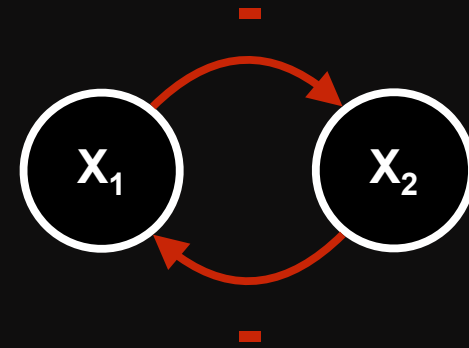
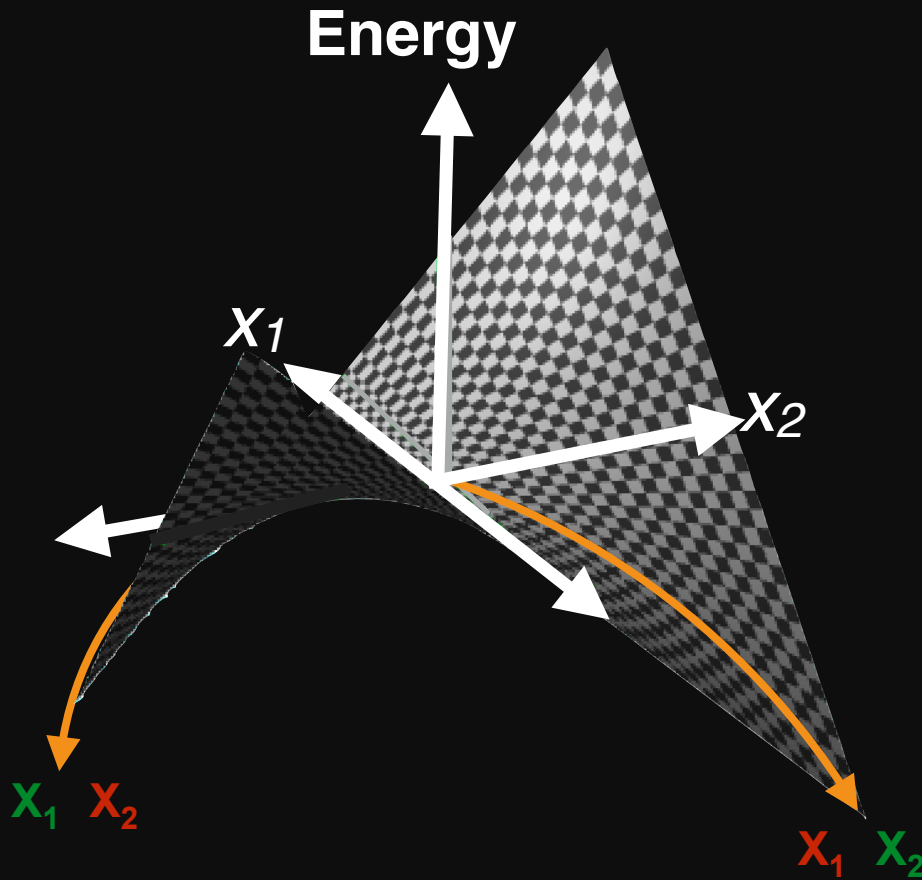
- Hard to visualize in high dimensions, so stick to 2.5-D...

# Energy Landscapes



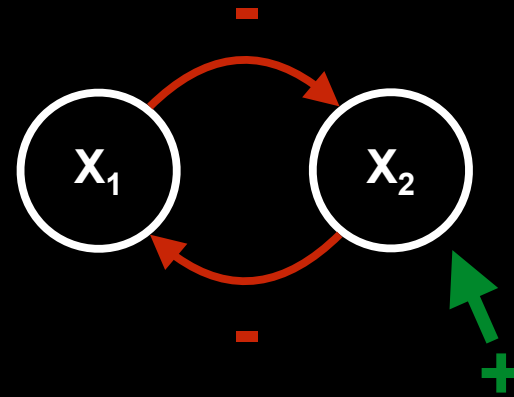
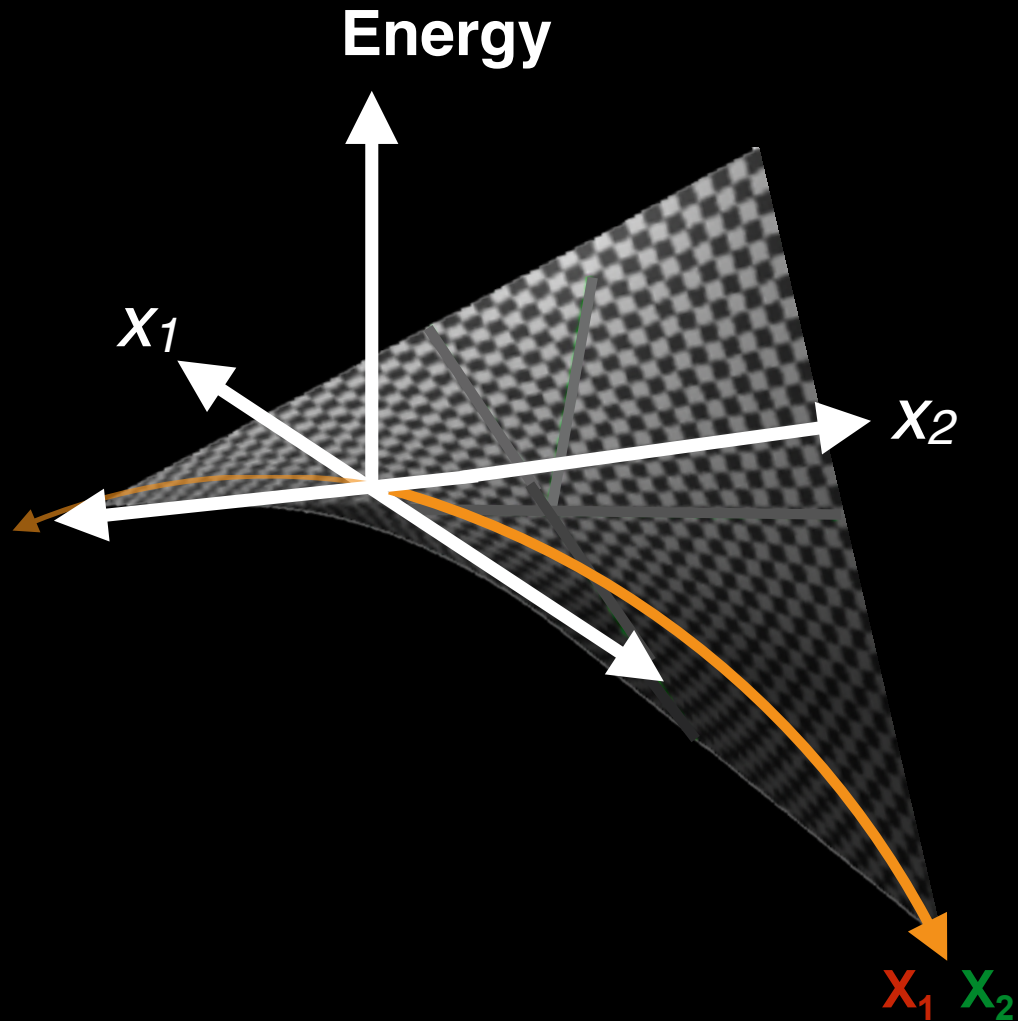
Mutually Excitatory units

# Energy Landscapes



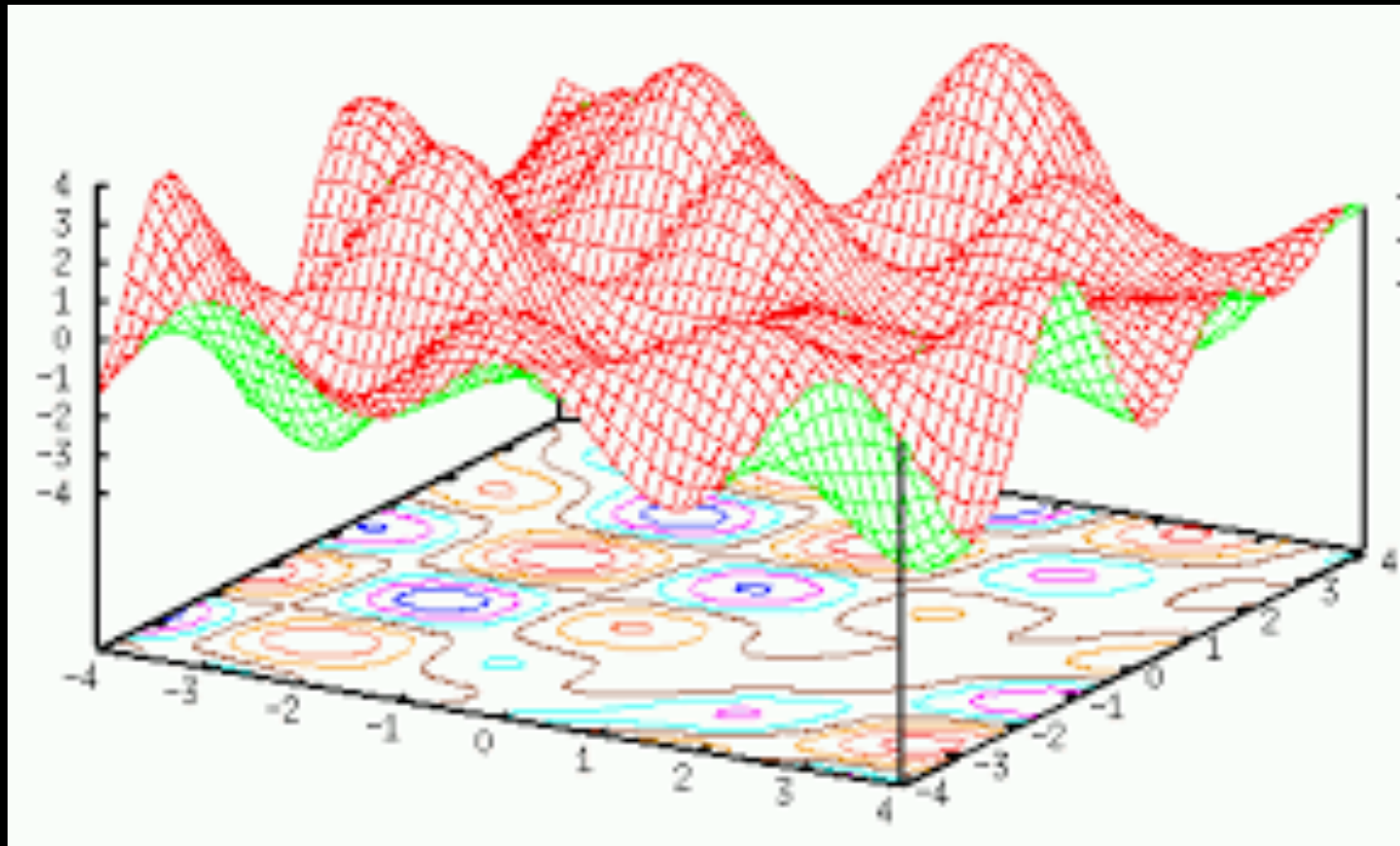
Mutually Inhibitory units

# Energy Landscapes



Mutually Inhibitory units  
and strong input to  $X_2$

# Energy Landscapes





# Minima

---

# Minima

---

- Minima define stable states of the system: **attractors**
  - “wells” in the energy landscape

# Minima

---

- Minima define stable states of the system: **attractors**
  - “wells” in the energy landscape
- **System will head to the nearest well: settling**
  - ball will roll downhill to the nearest well and stay there

# Minima

---

- Minima define stable states of the system: **attractors**
  - “wells” in the energy landscape
- System will head to the nearest well: **settling**
  - ball will roll downhill to the nearest well and stay there
- **Settling = perception** (*or retrieval*)
  - values of the units in that state reflect the properties of the perceptual interpretation (*or retrieved memory*)

# Minima

---

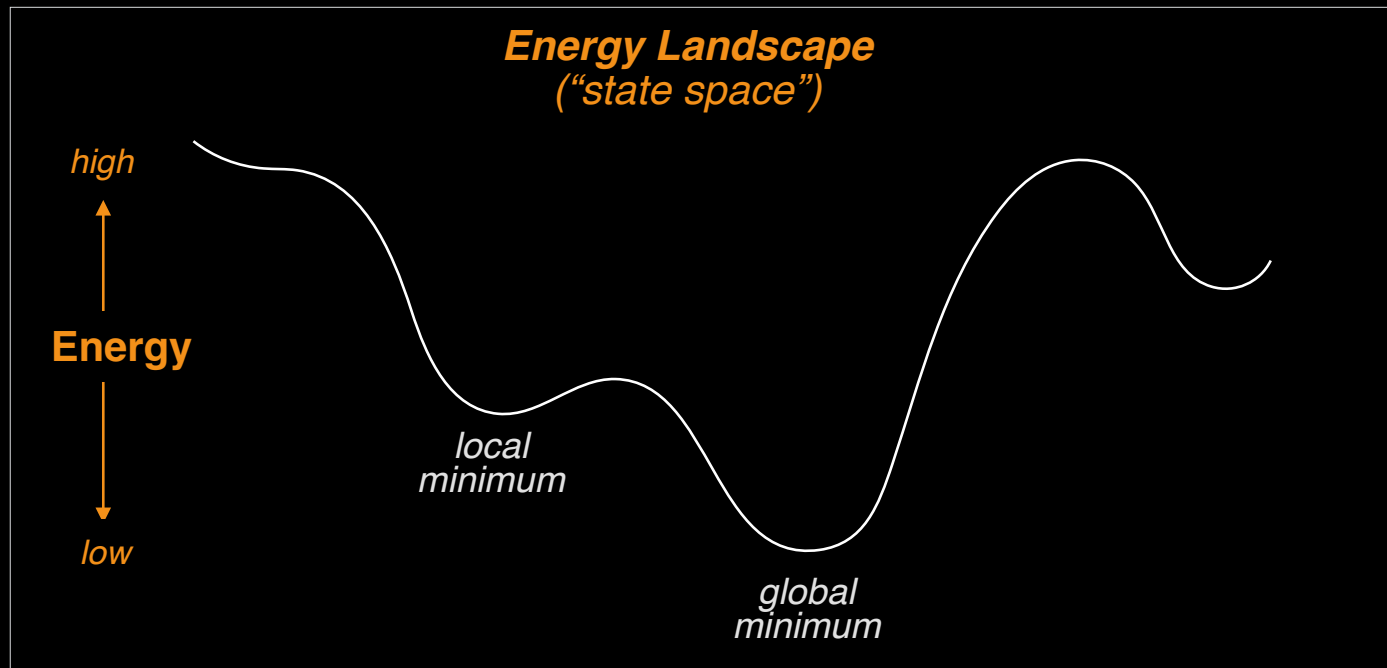
- Minima define stable states of the system: **attractors**
  - “wells” in the energy landscape
- System will head to the nearest well: **settling**
  - ball will roll downhill to the nearest well and stay there
- Settling = perception (*or retrieval*)
  - values of the units in that state reflect the properties of the perceptual interpretation (*or retrieved memory*)
- **No guarantee that nearest minimum is the best: local minima**
  - ball can get stuck in a shallow well before finding deepest one...

# Local Minima

---

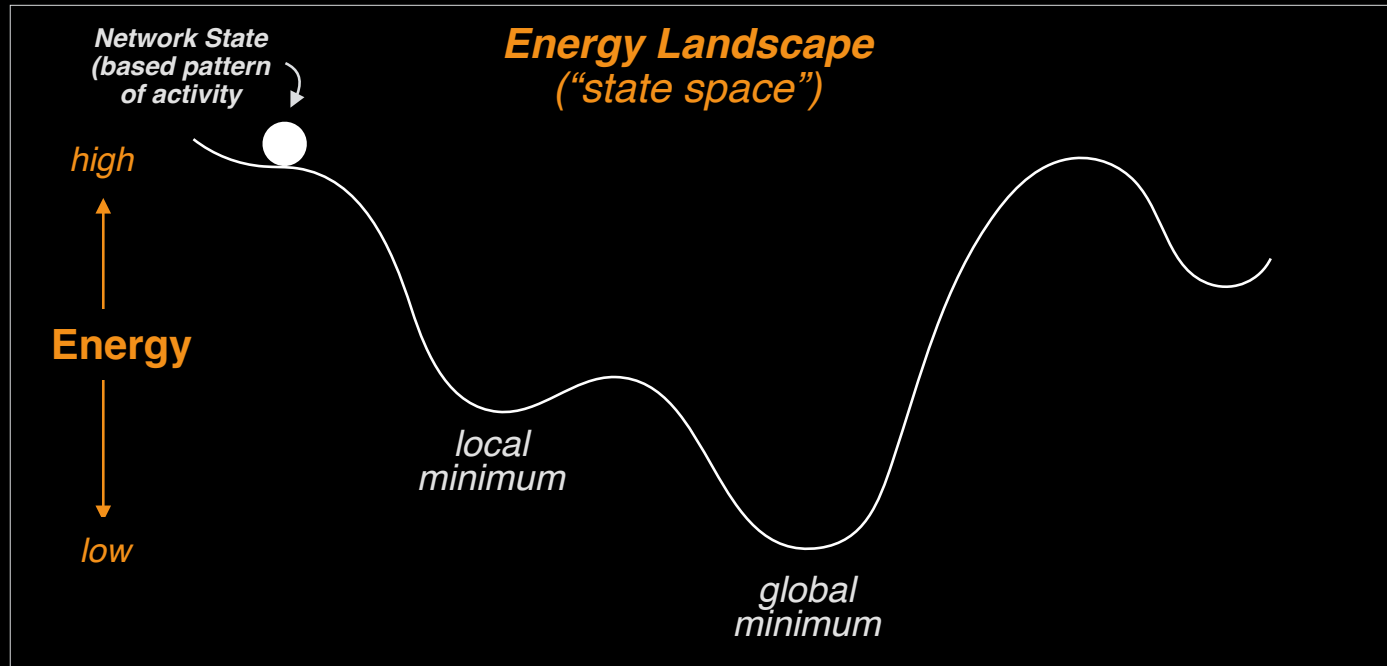
# Local Minima

---



# Local Minima

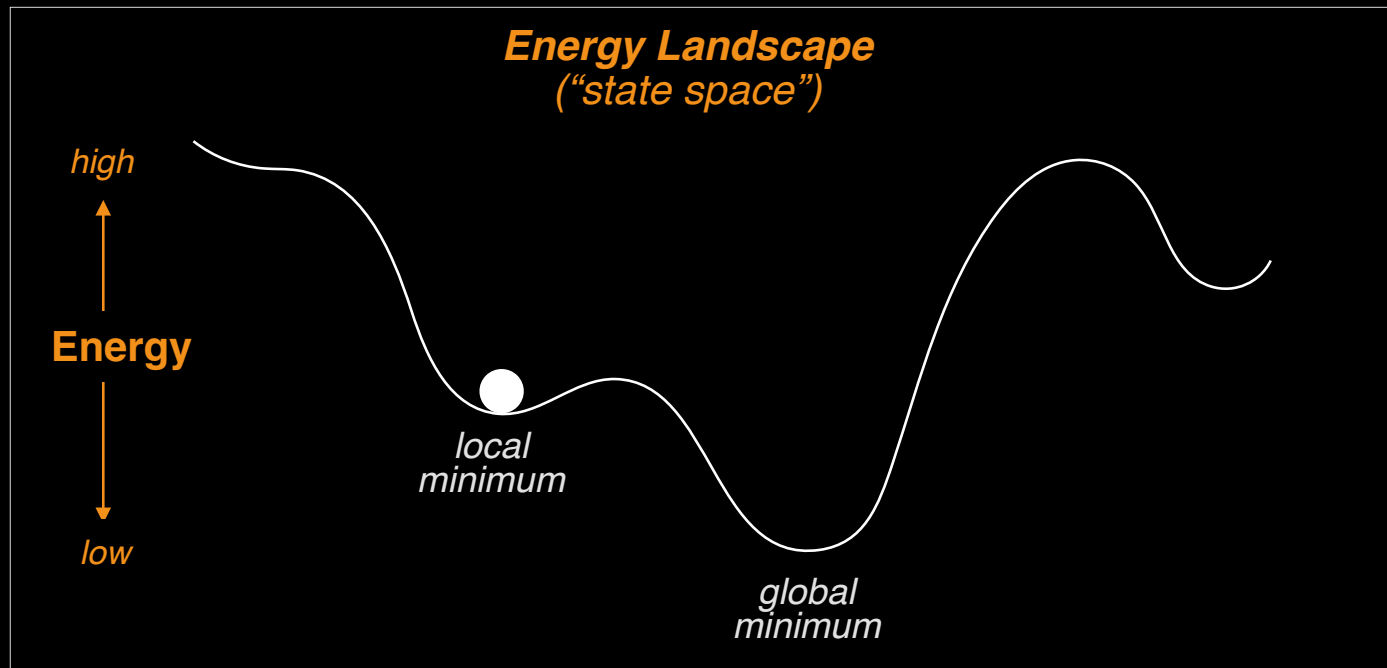
---





# Local Minima

---



# Local Minima

---

# Local Minima

---

- **Predisposing factors to getting stuck in local minima:**
  - Neighborhood effects (coalitions)
  - Binary units or large changes in activation (rapid updating)

# Local Minima

---

- Predisposing factors to getting stuck in local minima:
  - Neighborhood effects (coalitions)
  - Binary units or large changes in activation (rapid updating)

- **Avoiding local minima** (*“annealing”*):

- **Continuous activation values**

small adjustments in activity in each update prevents units from “committing themselves” before they feel more distant (global) influences

# Local Minima

---

- Predisposing factors to getting stuck in local minima:
  - Neighborhood effects (coalitions)
  - Binary units or large changes in activation (rapid updating)

- **Avoiding local minima** (*“annealing”*):

- **Continuous activation values**

small adjustments in activity in each update prevents units from “committing themselves” before they feel more distant (global) influences  
but it takes longer to settle

# Local Minima

---

- Predisposing factors to getting stuck in local minima:
  - Neighborhood effects (coalitions)
  - Binary units or large changes in activation (rapid updating)
- **Avoiding local minima (“annealing”):**
  - **Continuous activation values**
    - small adjustments in activity in each update prevents units from “committing themselves” before they feel more distant (global) influences
    - but it takes longer to settle
  - **Stochastic activation (noisy input)**

# Local Minima

---

- Predisposing factors to getting stuck in local minima:
  - Neighborhood effects (coalitions)
  - Binary units or large changes in activation (rapid updating)

- **Avoiding local minima (“annealing”):**

- **Continuous activation values**

- small adjustments in activity in each update prevents units from “committing themselves” before they feel more distant (global) influences
    - but it takes longer to settle

- **Stochastic activation (noisy input)**

- allows the system to “correct” itself...
    - can back out of blind alley, or literally “jump” out of a local minimum

# Local Minima

---

- Predisposing factors to getting stuck in local minima:
  - Neighborhood effects (coalitions)
  - Binary units or large changes in activation (rapid updating)

- **Avoiding local minima (“annealing”):**

- **Continuous activation values**

- small adjustments in activity in each update prevents units from “committing themselves” before they feel more distant (global) influences
    - but it takes longer to settle

- **Stochastic activation (noisy input)**

- allows the system to “correct” itself...
    - can back out of blind alley, or literally “jump” out of a local minimum
    - but less predictable



# Local Minima

---

- Predisposing factors to getting stuck in local minima:
  - Neighborhood effects (coalitions)
  - Binary units or large changes in activation (rapid updating)
- **Avoiding local minima (“annealing”):**
  - **Continuous activation values**
    - small adjustments in activity in each update prevents units from “committing themselves” before they feel more distant (global) influences
    - but it takes longer to settle
  - **Stochastic activation (noisy input)**
    - allows the system to “correct” itself...
    - can back out of blind alley, or literally “jump” out of a local minimum
    - but less predictable
  - **Temperature modulates the effects of both...**

# Effects of Temperature

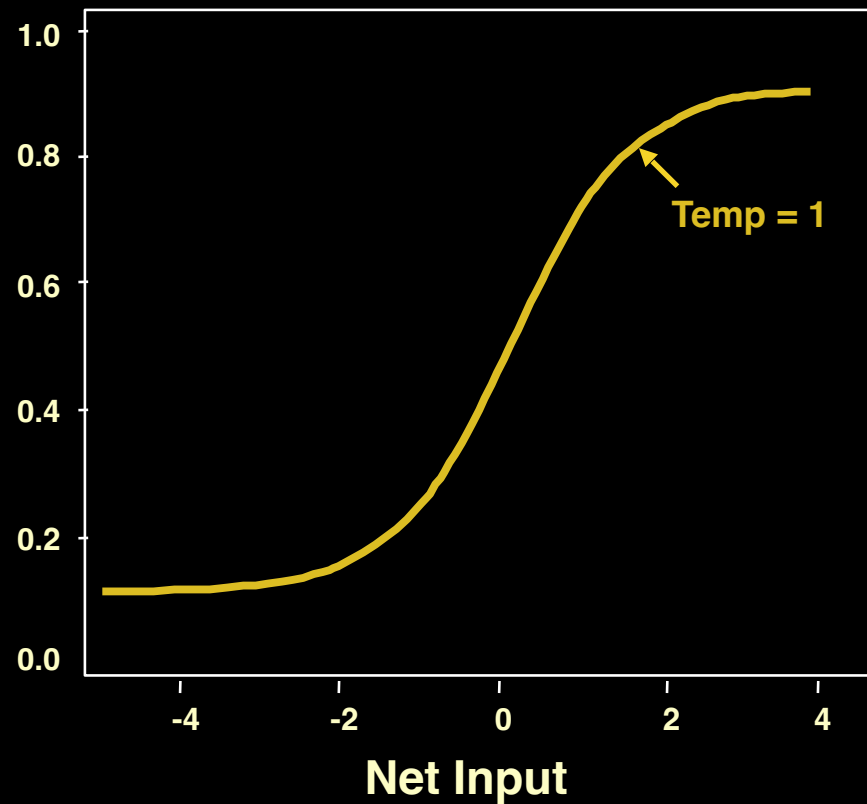
$$\frac{1}{1 + e^{-\frac{\text{net input}}{\text{Temp}}}}$$

**Activity**

*(Hopfield Net, 1984)*

**Probability of firing**

*(Boltzmann Machine)*



# Effects of Temperature

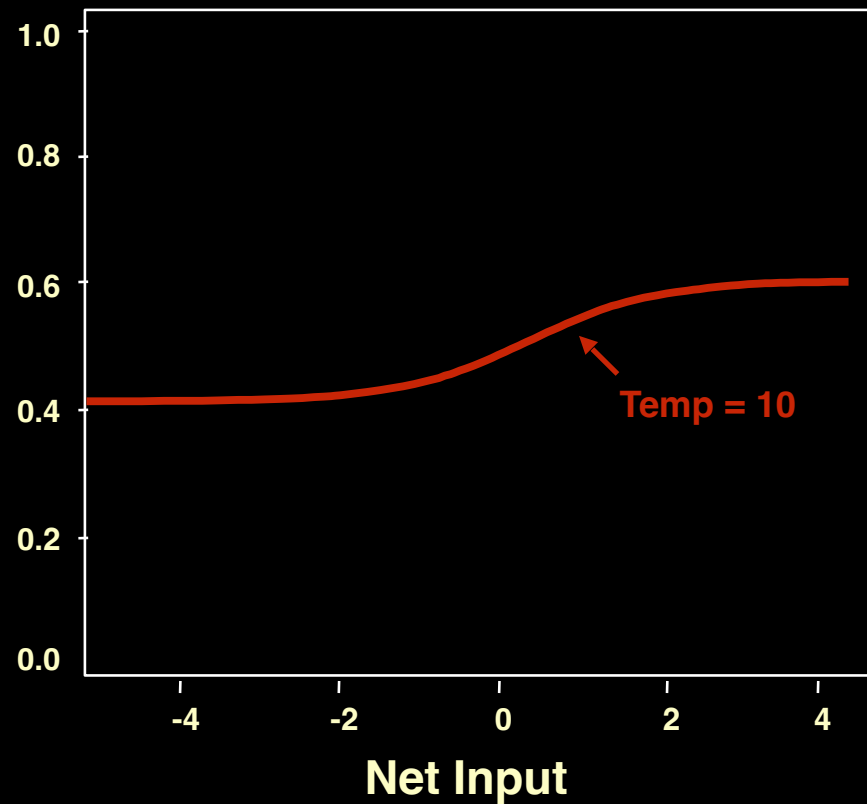
$$\frac{1}{1 + e^{-\frac{\text{net input}}{\text{Temp}}}}$$

**Activity**

*(Hopfield Net, 1984)*

**Probability of firing**

*(Boltzmann Machine)*



# Effects of Temperature

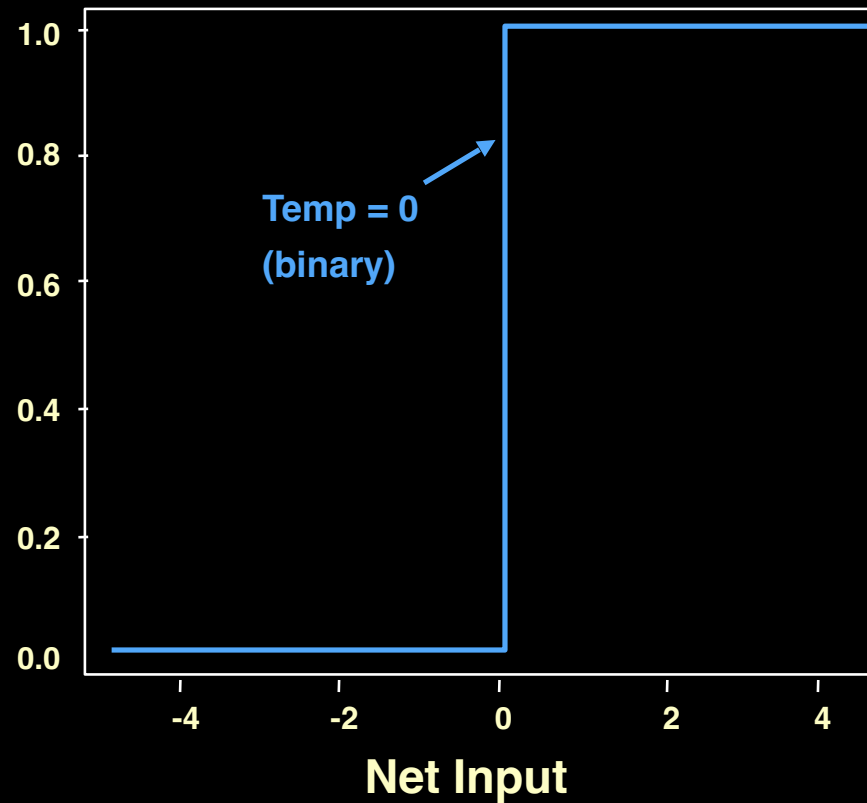
$$\frac{1}{1 + e^{-\frac{\text{net input}}{\text{Temp}}}}$$

**Activity**

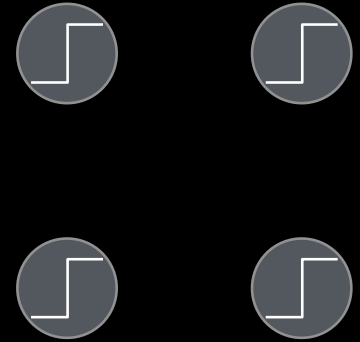
*(Hopfield Net, 1984)*

**Probability of firing**

*(Boltzmann Machine)*



# Hopfield (1982) Network



# Hopfield (1982) Network

---

- **Critical assumptions**

- **Non-linear (binary) units**

- ◆ forces them to make “decisions:”

- categorize an input as reflecting one memory or another;
      - vs. linear systems that represent *graded* blends of options



# Hopfield (1982) Network

- **Critical assumptions**

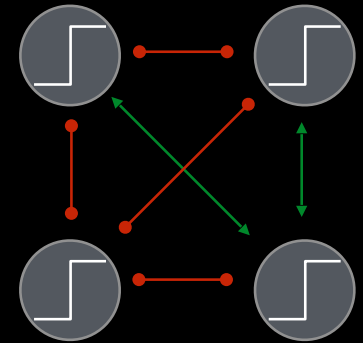
- **Non-linear (binary) units**

- ◆ forces them to make “decisions:”

- categorize an input as reflecting one memory or another;
      - vs. linear systems that represent *graded* blends of options

- **Recurrent connections**

- ◆ provides basis for settling dynamics and attractors
    - ◆ **symmetric**: required for analysis, but not critical for function



# Hopfield (1982) Network

- **Critical assumptions**

- **Non-linear (binary) units**

- ◆ forces them to make “decisions:”

- categorize an input as reflecting one memory or another;
      - vs. linear systems that represent *graded* blends of options

- **Recurrent connections**

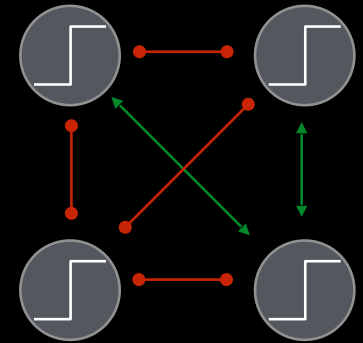
- ◆ provides basis for settling dynamics and attractors

- ◆ symmetric: required for analysis, but not critical for function

- **Asynchronous updating**

- ◆ biologically plausible

- ◆ insures symmetry breaking, avoids “see-sawing” (pseudo-stochasticity)





# Hopfield (1982) Network

- **Critical assumptions**

- **Non-linear (binary) units**

- ◆ forces them to make “decisions:”

- categorize an input as reflecting one memory or another;
      - vs. linear systems that represent *graded* blends of options

- **Recurrent connections**

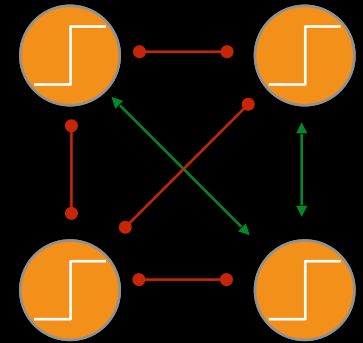
- ◆ provides basis for settling dynamics and attractors

- ◆ symmetric: required for analysis, but not critical for function

- **Asynchronous updating**

- ◆ biologically plausible

- ◆ insures symmetry breaking, avoids “see-sawing” (pseudo-stochasticity)



# Hopfield (1982) Network

- **Critical assumptions**

- **Non-linear (binary) units**

- ◆ forces them to make “decisions:”
      - categorize an input as reflecting one memory or another;
      - vs. linear systems that represent *graded* blends of options

- **Recurrent connections**

- ◆ provides basis for settling dynamics and attractors
    - ◆ symmetric: required for analysis, but not critical for function

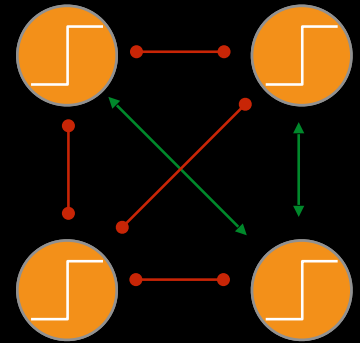
- **Asynchronous updating**

- ◆ biologically plausible
    - ◆ insures symmetry breaking, avoids “see-sawing” (pseudo-stochasticity)

- **Additional assumptions**

- **Deterministic**

- ◆ each unit does exactly what it is “told” by its neighbors (no noise)



# Hopfield (1982) Network

- **Critical assumptions**

- **Non-linear (binary) units**

- ◆ forces them to make “decisions:”
      - categorize an input as reflecting one memory or another;
      - vs. linear systems that represent *graded* blends of options

- **Recurrent connections**

- ◆ provides basis for settling dynamics and attractors
    - ◆ symmetric: required for analysis, but not critical for function

- **Asynchronous updating**

- ◆ biologically plausible
    - ◆ insures symmetry breaking, avoids “see-sawing” (pseudo-stochasticity)

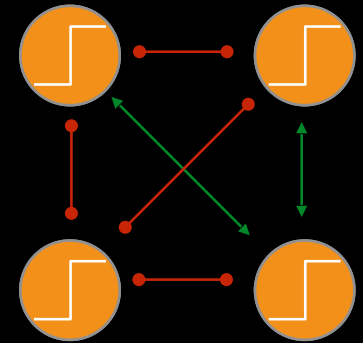
- **Additional assumptions**

- **Deterministic**

- ◆ each unit does exactly what it is “told” by its neighbors (no noise)

- **No self-connections**

- ◆ no “memory; each unit governed entirely by sampling its neighbors



# Hopfield (1982) Network

- **Critical assumptions**

- **Non-linear (binary) units**

- ◆ forces them to make “decisions:”
      - categorize an input as reflecting one memory or another;
      - vs. linear systems that represent *graded* blends of options

- **Recurrent connections**

- ◆ provides basis for settling dynamics and attractors
    - ◆ symmetric: required for analysis, but not critical for function

- **Asynchronous updating**

- ◆ biologically plausible
    - ◆ insures symmetry breaking, avoids “see-sawing” (pseudo-stochasticity)

- **Additional assumptions**

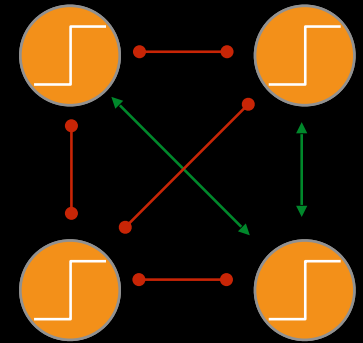
- **Deterministic**

- ◆ each unit does exactly what it is “told” by its neighbors (no noise)

- **No self-connections**

- ◆ no “memory”; each unit governed entirely by sampling its neighbors

- **Otherwise, fully interconnected**



# **Contributions**

---

# Contributions

---

- **Demonstration of how a neural network can compute:**
  - **Biologically-inspired assumptions**

# Contributions

---

- **Demonstration of how a neural network can compute:**
  - Biologically-inspired assumptions
  - **Percepts (*memories*) can be represented (*stored*) as minima (attractors)**

# Contributions

---

- **Demonstration of how a neural network can compute:**
  - Biologically-inspired assumptions
  - Percepts (*memories*) can be represented (*stored*) as minima (attractors)
  - **Algorithm for producing them using Hebbian learning**



# Contributions

---

- **Demonstration of how a neural network can compute:**
  - Biologically-inspired assumptions
  - Percepts (*memories*) can be represented (*stored*) as minima (attractors)
  - Algorithm for producing them using Hebbian learning
  - **Emergent properties:**
    - ◆ Gestalt categorization
    - ◆ Content-addressability
    - ◆ Dynamics

# Contributions

---

- **Demonstration of how a neural network can compute:**
  - Biologically-inspired assumptions
  - Percepts (*memories*) can be represented (*stored*) as minima (attractors)
  - Algorithm for producing them using Hebbian learning
  - Emergent properties:
    - ♦ Gestalt categorization
    - ♦ Content-addressability
    - ♦ Dynamics
- **Capacity:**
  - roughly 15% no. of units, before minima become too narrow / shallow

# Contributions

---

- **Demonstration of how a neural network can compute:**
  - Biologically-inspired assumptions
  - Percepts (*memories*) can be represented (*stored*) as minima (attractors)
  - Algorithm for producing them using Hebbian learning
  - Emergent properties:
    - ◆ Gestalt categorization
    - ◆ Content-addressability
    - ◆ Dynamics
- **Capacity:**
  - roughly 15% no. of units, before minima become too narrow / shallow
- **Connection to statistical mechanics (*Ising model*):**

# Contributions

---

- **Demonstration of how a neural network can compute:**
  - Biologically-inspired assumptions
  - Percepts (*memories*) can be represented (*stored*) as minima (attractors)
  - Algorithm for producing them using Hebbian learning
  - Emergent properties:
    - ♦ Gestalt categorization
    - ♦ Content-addressability
    - ♦ Dynamics
- **Capacity:**
  - roughly 15% no. of units, before minima become too narrow / shallow
- **Connection to statistical mechanics (*Ising model*):**
  - Can think about neural networks in terms of state-space (or “phase space”) dynamics of energy minimization & annealing

# Contributions

---

- **Demonstration of how a neural network can compute:**
  - Biologically-inspired assumptions
  - Percepts (*memories*) can be represented (*stored*) as minima (attractors)
  - Algorithm for producing them using Hebbian learning
  - Emergent properties:
    - ♦ Gestalt categorization
    - ♦ Content-addressability
    - ♦ Dynamics
- **Capacity:**
  - roughly 15% no. of units, before minima become too narrow / shallow
- **Connection to statistical mechanics (*Ising model*):**
  - Can think about neural networks in terms of state-space (or “phase space”) dynamics of energy minimization & annealing
  - Energy landscapes describe network dynamics

# Contributions

---

- **Demonstration of how a neural network can compute:**
  - Biologically-inspired assumptions
  - Percepts (*memories*) can be represented (*stored*) as minima (attractors)
  - Algorithm for producing them using Hebbian learning
  - Emergent properties:
    - ♦ Gestalt categorization
    - ♦ Content-addressability
    - ♦ Dynamics
- **Capacity:**
  - roughly 15% no. of units, before minima become too narrow / shallow
- **Connection to statistical mechanics (*Ising model*):**
  - Can think about neural networks in terms of state-space (or “phase space”) dynamics of energy minimization & annealing
  - Energy landscapes describe network dynamics
- **Connection to quantum computing (*Hamiltonian dynamics*)**

# Comparison of Models

| <b>Model</b>  | <b>Activation</b> | <b>Updating</b>      | <b>Connections</b> |
|---|-------------------|----------------------|--------------------|
| <b>Hopfield (1982)</b>                              | <b>Binary</b>     | <b>Deterministic</b> | <b>Symmetric</b>   |
| <b>Hopfield (1984)</b>                              | <b>Continuous</b> | <b>Deterministic</b> | <b>Symmetric</b>   |
| <b>Boltzmann Machine</b>                            | <b>Binary</b>     | <b>Stochastic</b>    | <b>Asymmetric</b>  |
| <b>Interactive Activation and Competition (IAC)</b> | <b>Continuous</b> | <b>Deterministic</b> | <b>Symmetric</b>   |
| <b>Leaky Competing Accumulator (LCA)</b>            | <b>Continuous</b> | <b>Stochastic</b>    | <b>Asymmetric</b>  |

# **Comparison of Models**

---

- **To what extent are setting dynamics psychologically/ neurally plausible?**



# **Comparison of Models**

---

- **How can such models be used to account for empirical data...**

# **Interactive Activation Model of Word Perception**

---

**McClelland & Rumelhart, 1981**

# **Interactive Activation Model of Word Perception**

---

**McClelland & Rumelhart, 1981**

- **Word superiority effect:**
  - **Faster to recognize a letter in the context of a word than alone**

# Interactive Activation Model of Word Perception

McClelland & Rumelhart, 1981

- **Word superiority effect:**

- Faster to recognize a letter in the context of a word than alone

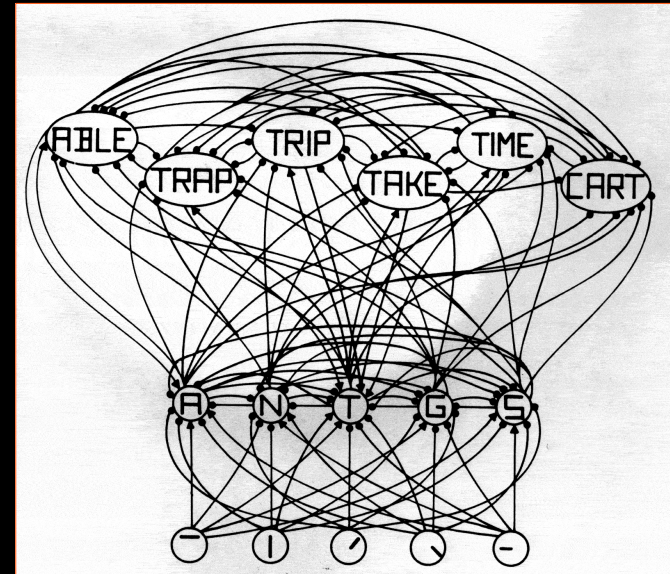
- **IAC model:**

- Accounts for empirical findings regarding word perception:

- ♦ frequency effects
- ♦ neighborhood effects
- ♦ word superiority effects

- Predicted new perceptual phenomena

- Landmark in formal modeling of complex psychological phenomena using connectionist architecture



# Interactive Activation Model of Word Perception

McClelland & Rumelhart, 1981

- **Word superiority effect:**

- Faster to recognize a letter in the context of a word than alone

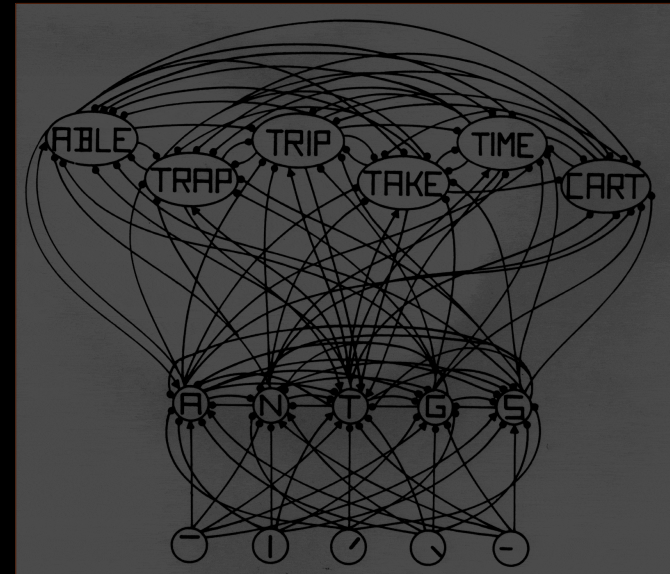
- **IAC model:**

- Accounts for empirical findings regarding word perception:

- ♦ frequency effects
- ♦ neighborhood effects
- ♦ word superiority effects

- Predicted new perceptual phenomena

- Landmark in formal modeling of complex psychological phenomena using connectionist architecture



- **Will come back to this under section on language**