Integration and Perceptual Decision Making

Decision Making
(writ “small”)

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- What are the *dynamics* that underlie these decisions… and how are the *controlled*?

Outline

- **Simple behavioral task:**
  - **Two alternative forced choice (2AFC) decision task**

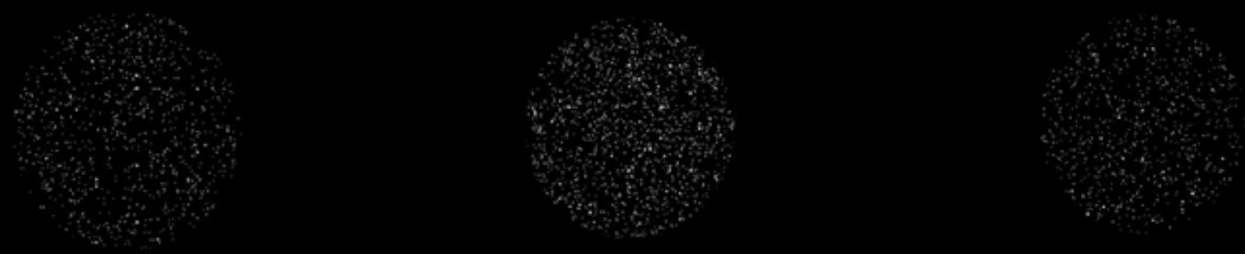
- **Current "state of play:"**
  - **Behavioral findings**
  - *(Some informative)* **neurobiological findings**
  - **PDP models**

- **Formal analysis:**
  - **Drift diffusion model (DDM)**

  Sets the stage for:
  - **Control as optimization**

Two Alternative Forced Choice Task

Are the dots moving left or right?



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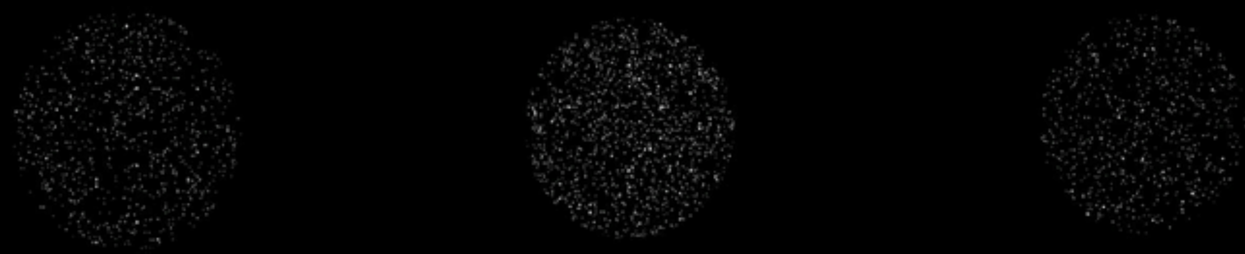

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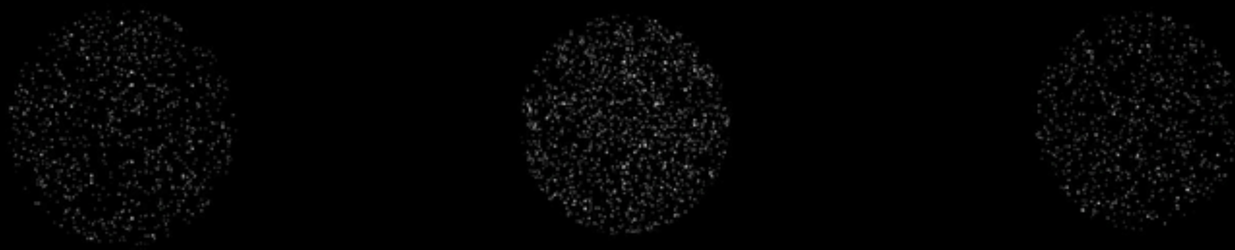

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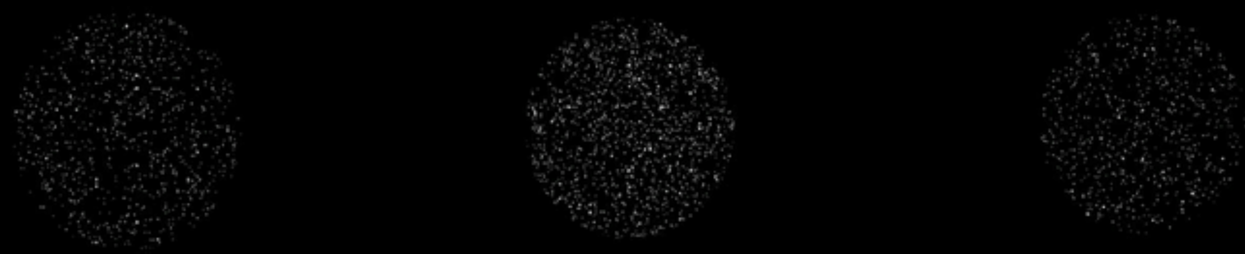

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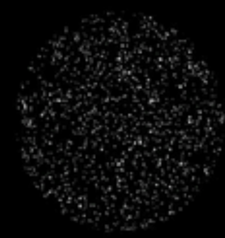

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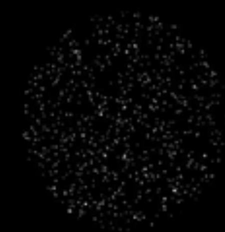

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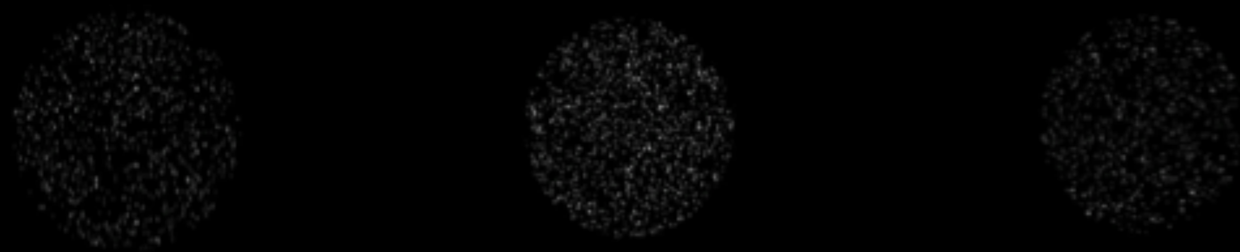
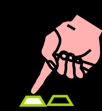


High coherence ➡ High coherence ➡ Low coherence ➡



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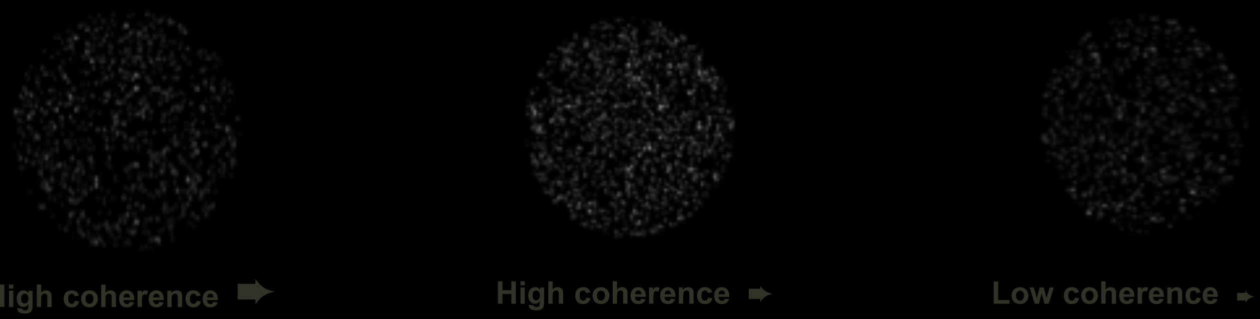

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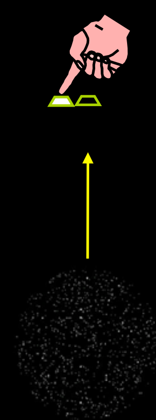

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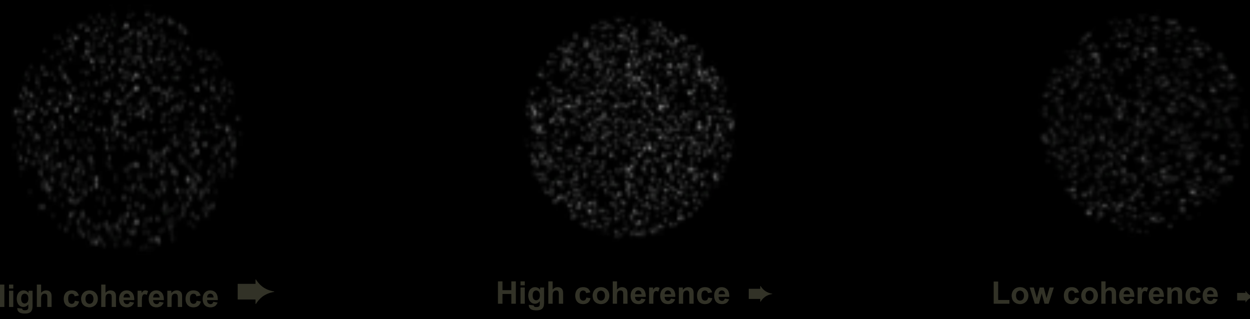

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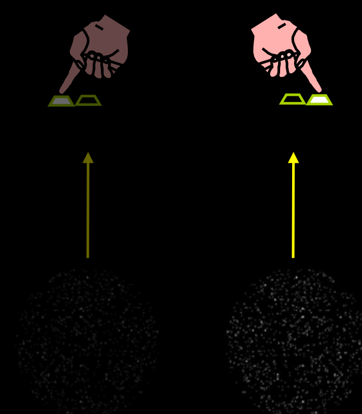

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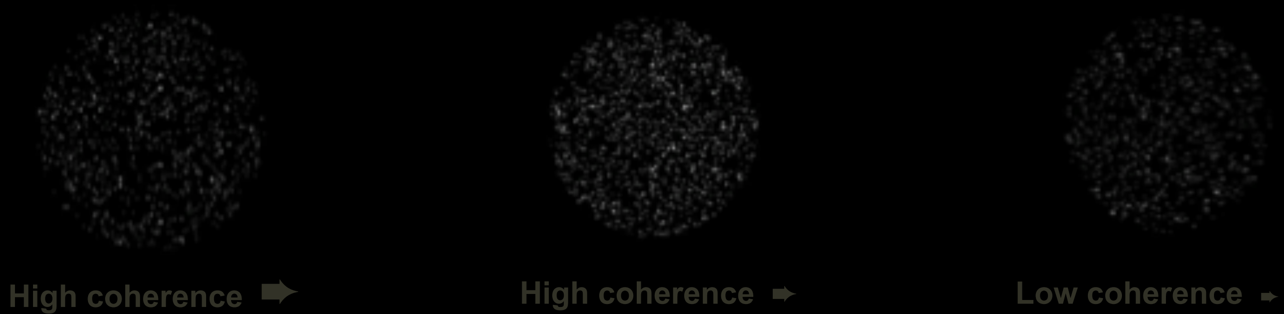

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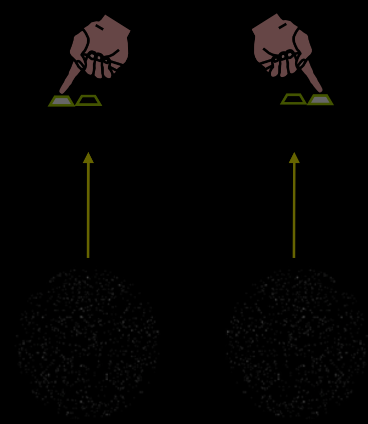
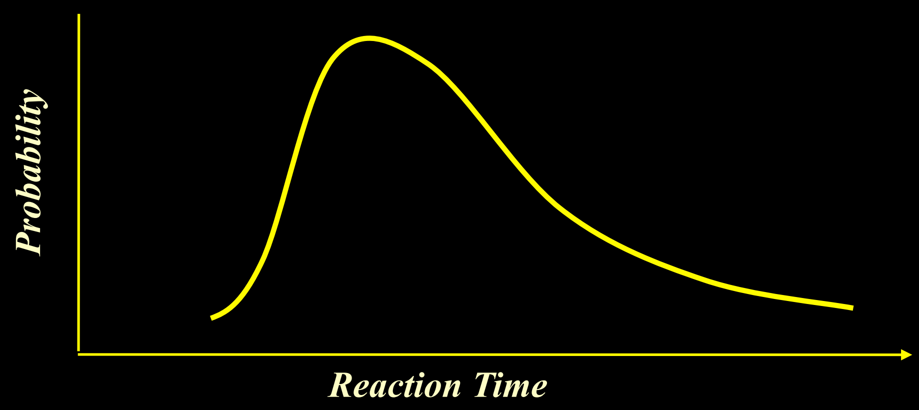


Measure reaction time (RT) and accuracy

Behavioral Findings



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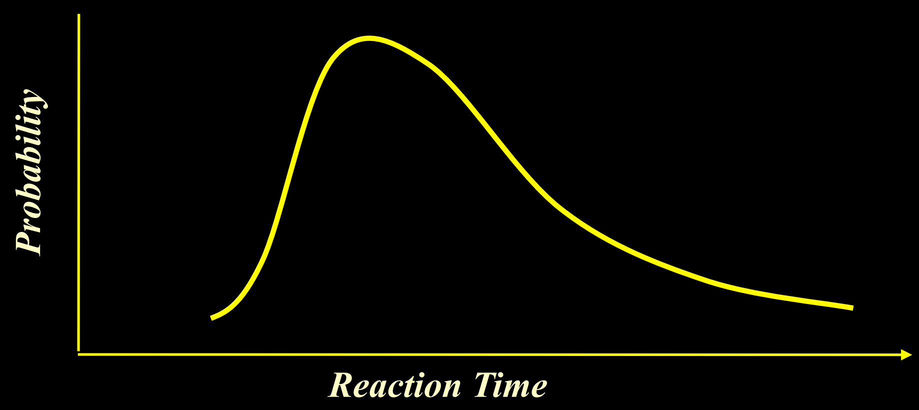

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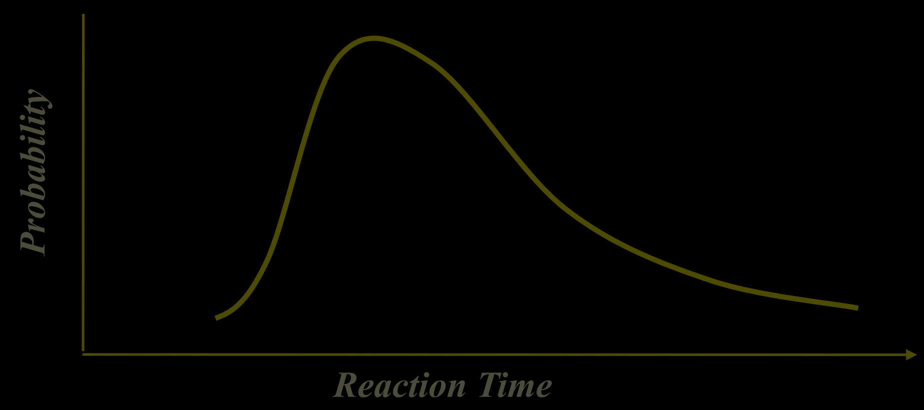
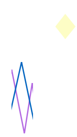


- Speed/accuracy tradeoff:
  - – faster response → less accurate
  - – more accurate → slower response

Neural Findings



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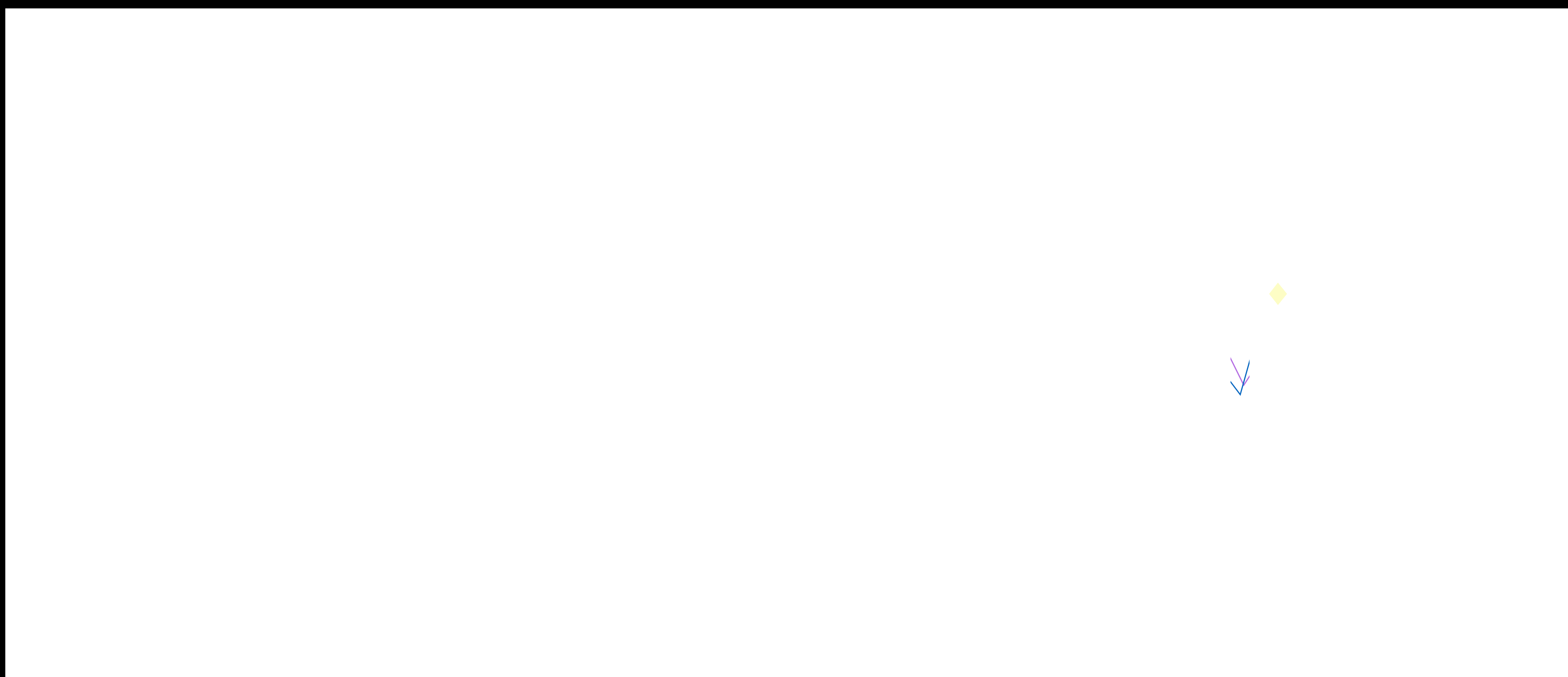

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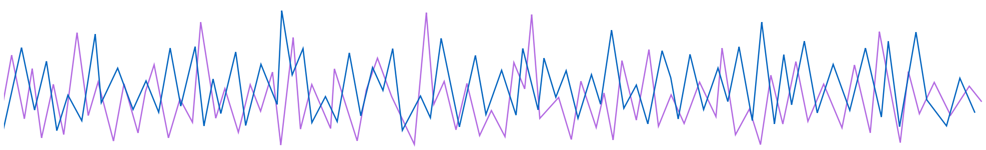

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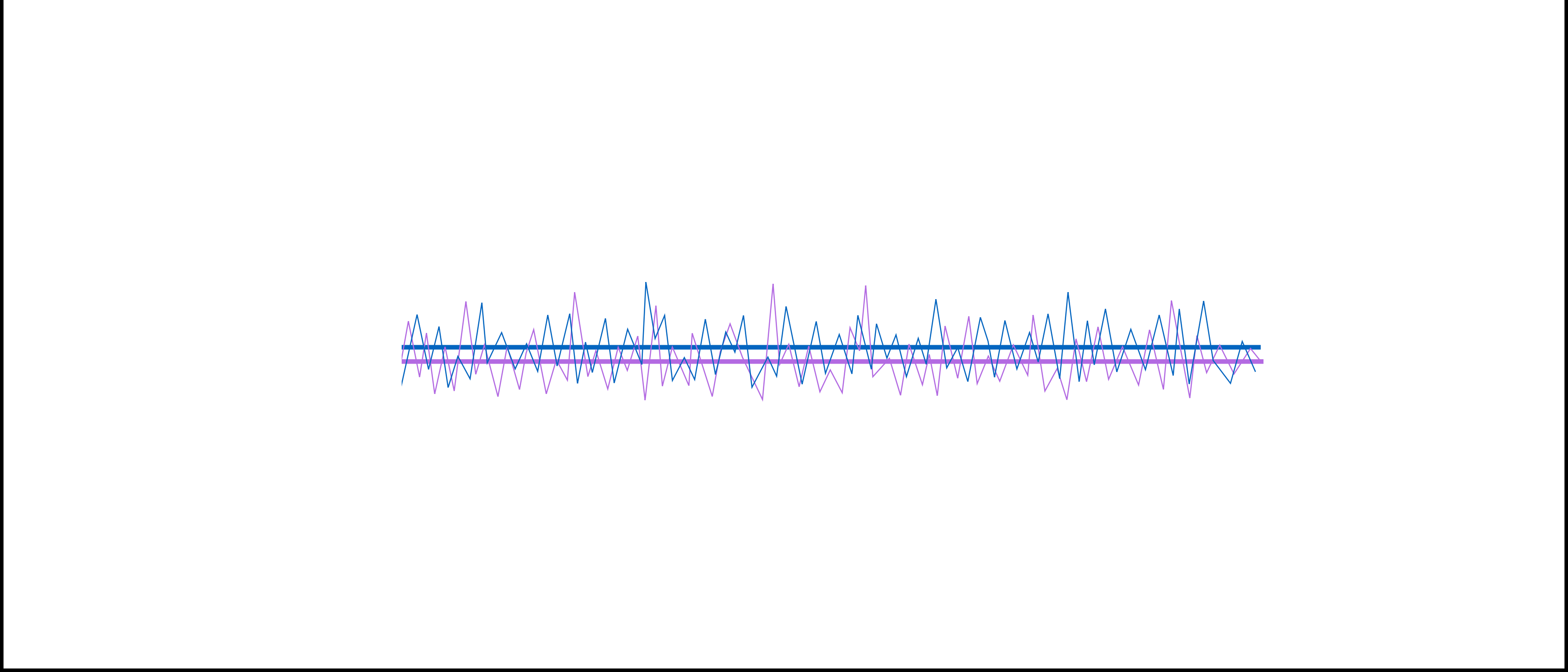

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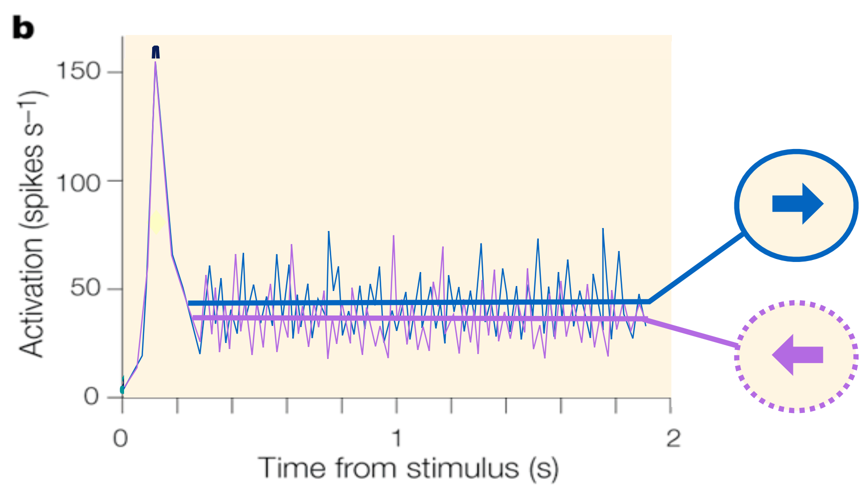
Neural Findings

Motion Detection:

Area MT
(temporal cortex)

motion sensitive
visual cortex



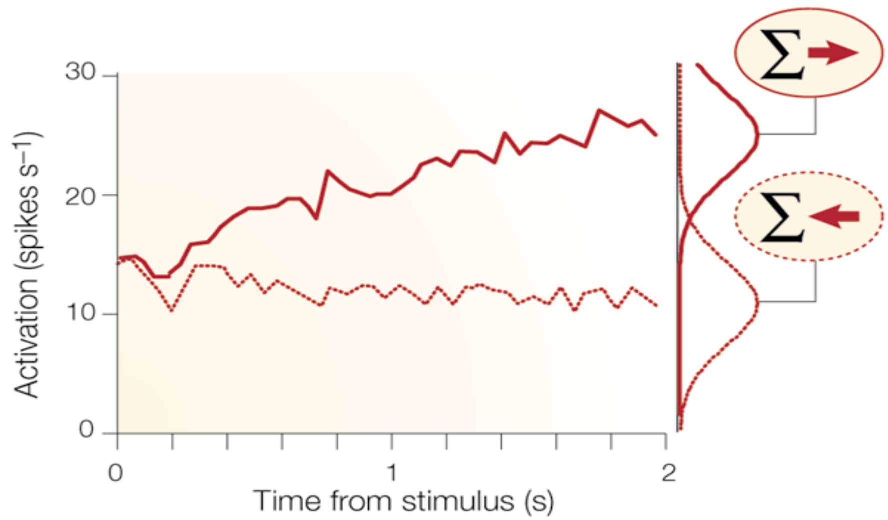
Shadlen & Newsome, 1998

Neural Findings

Integration:

Areas LIP
(parietal cortex)
and SEF
(supplementary eye fields):

control of
eye movements



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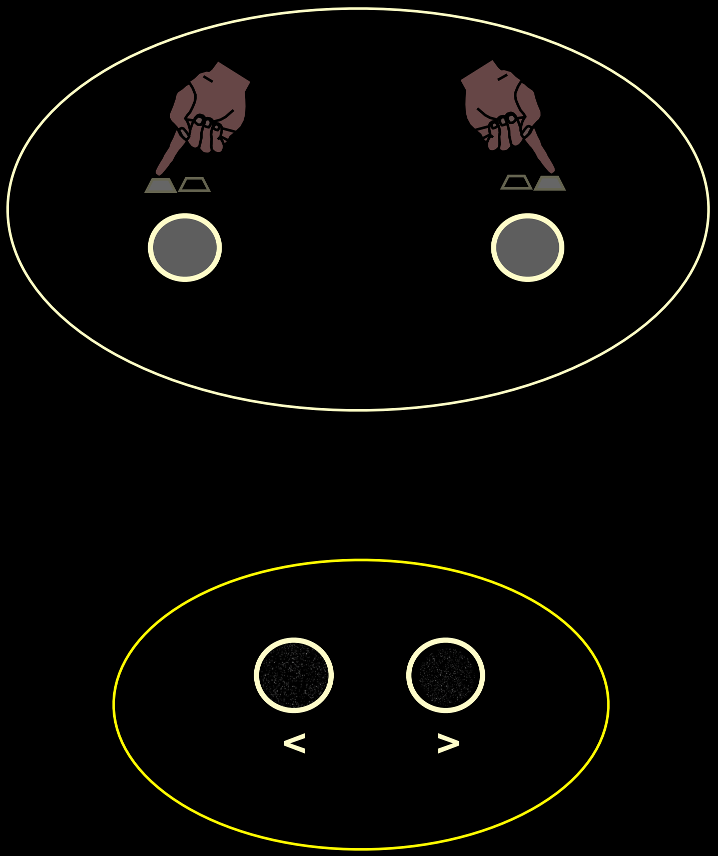
Simple PDP Model of Two Alternative Decision Task



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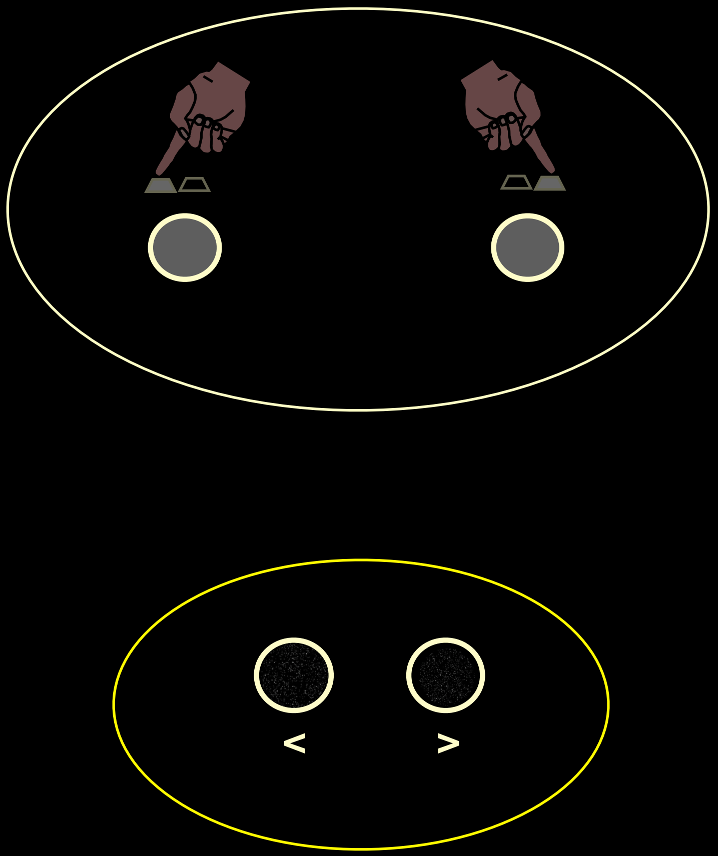

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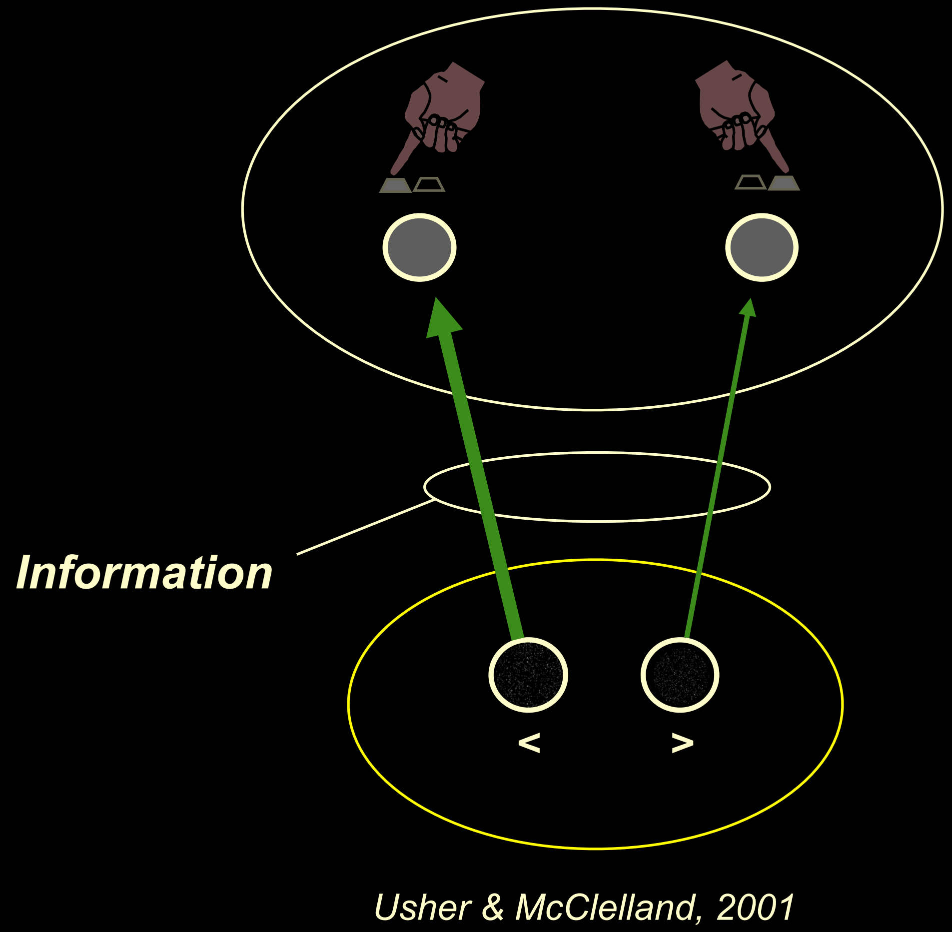

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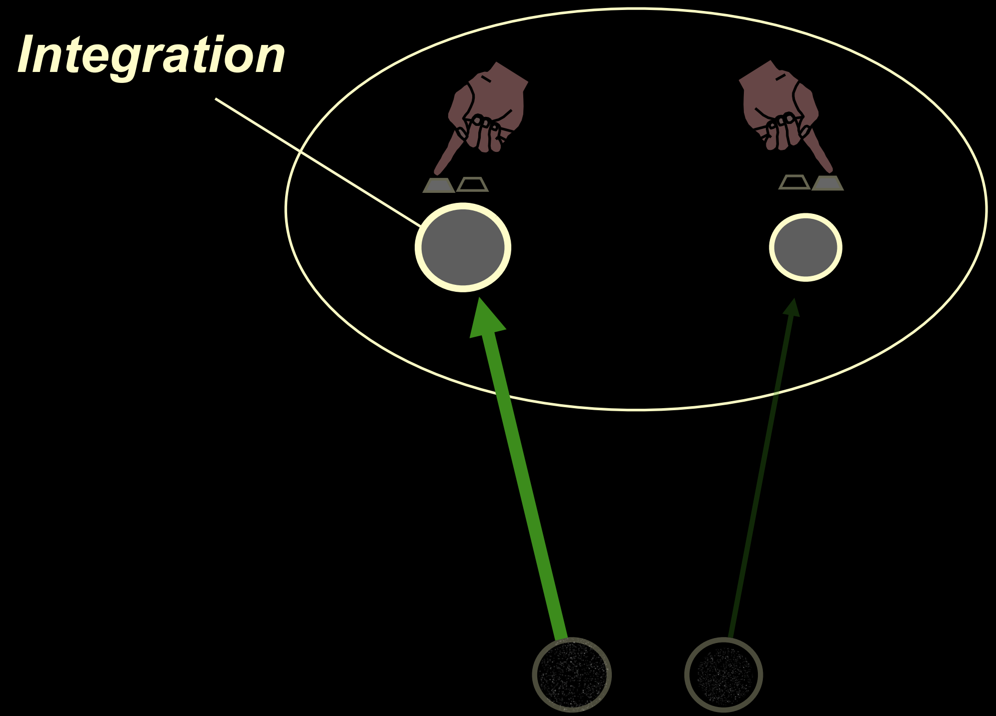

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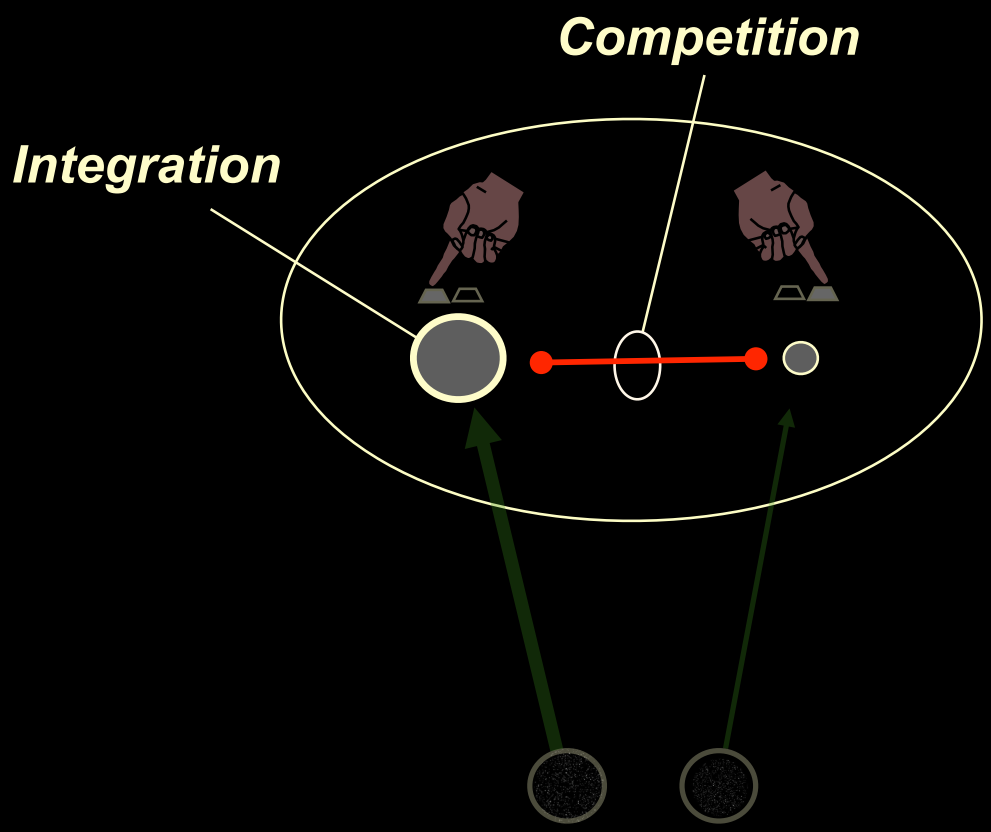

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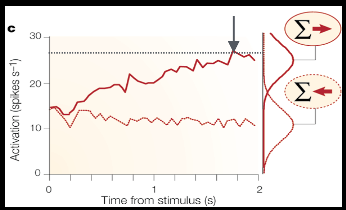

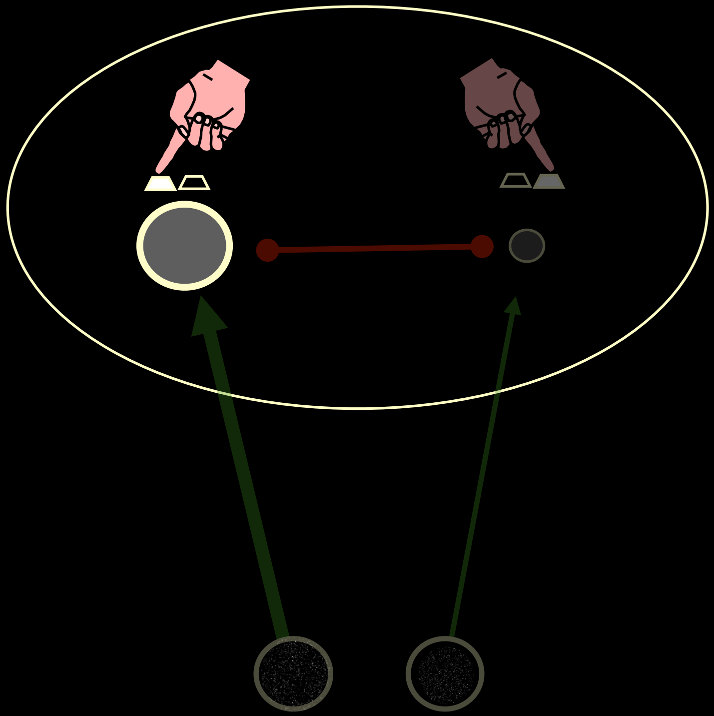

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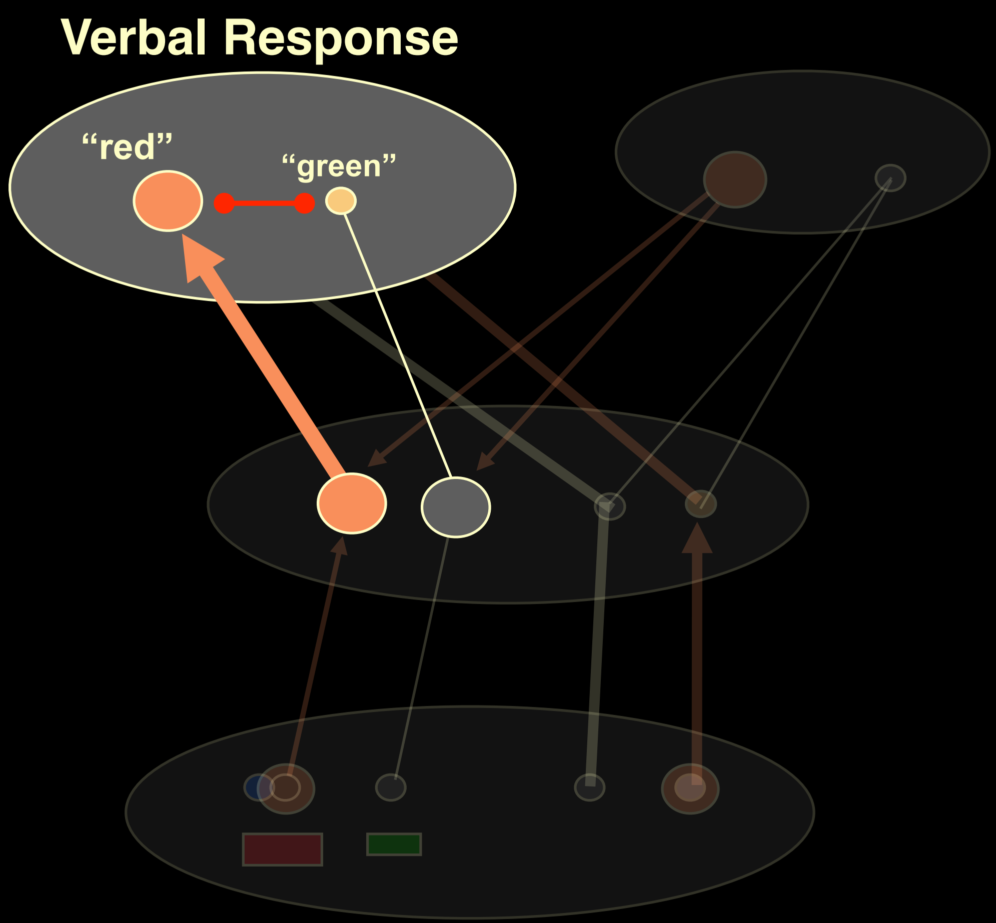

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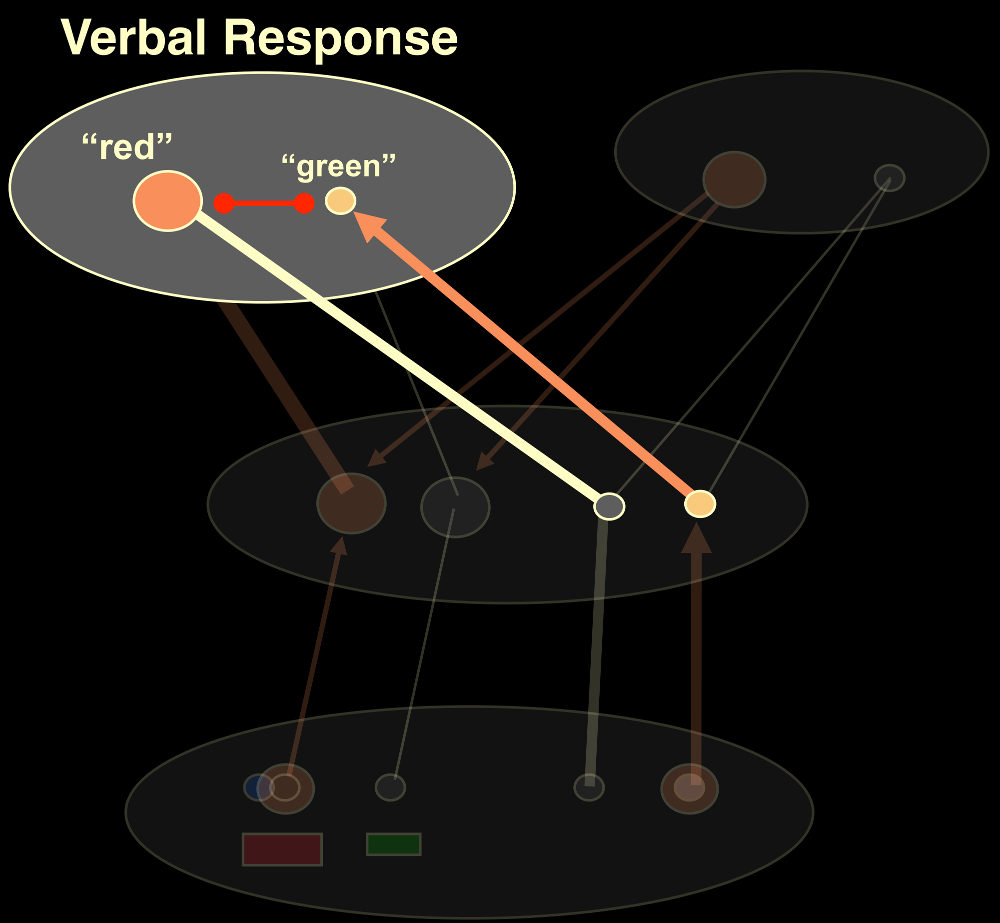

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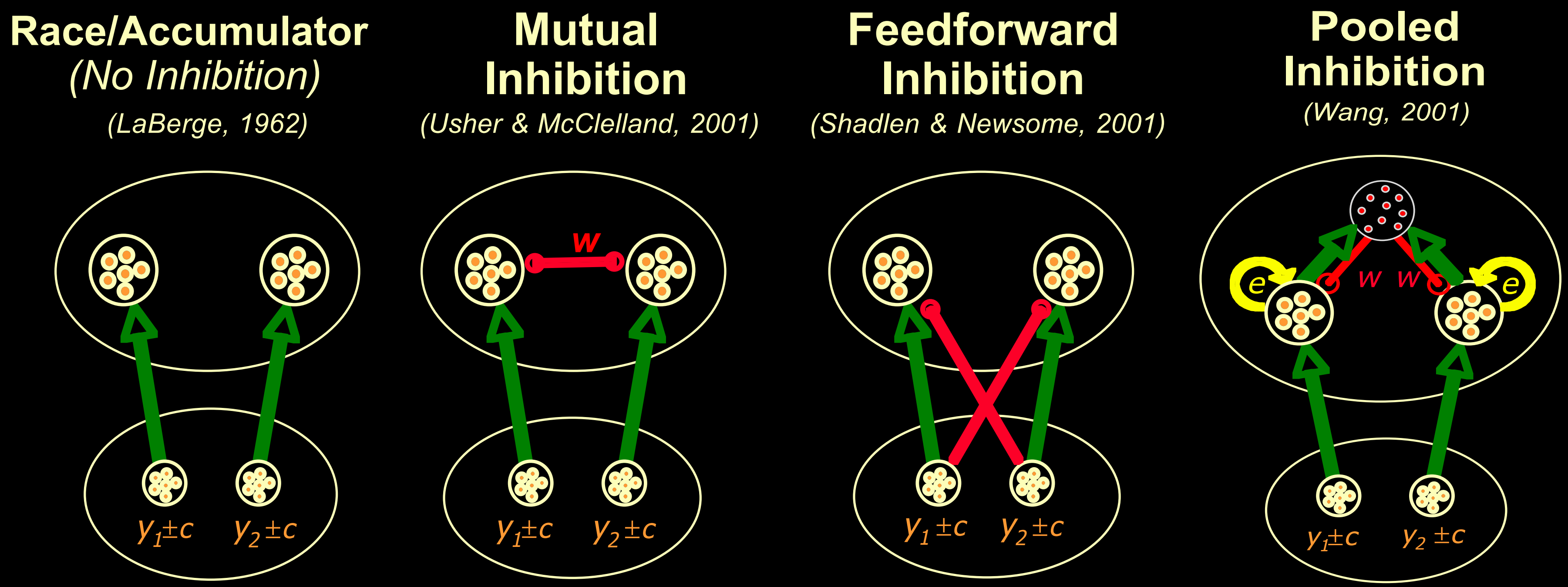
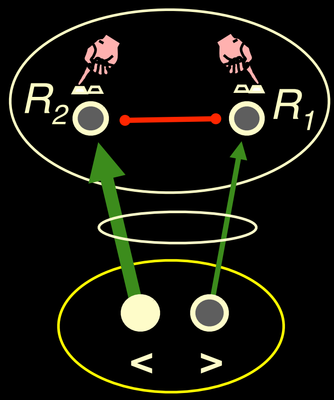


Simplification & Analysis

Step 1: Construct geometric representation of model's behavior



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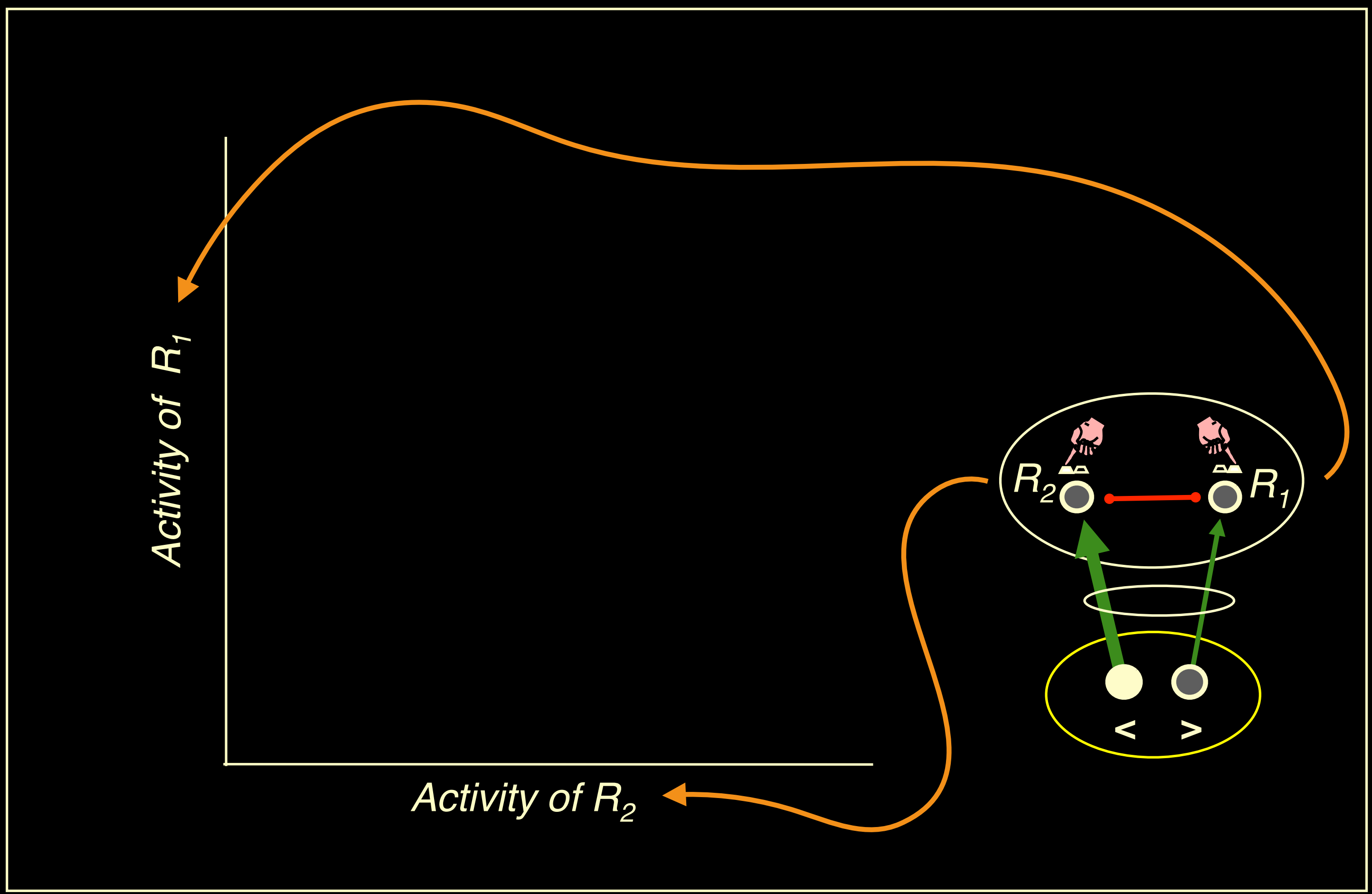

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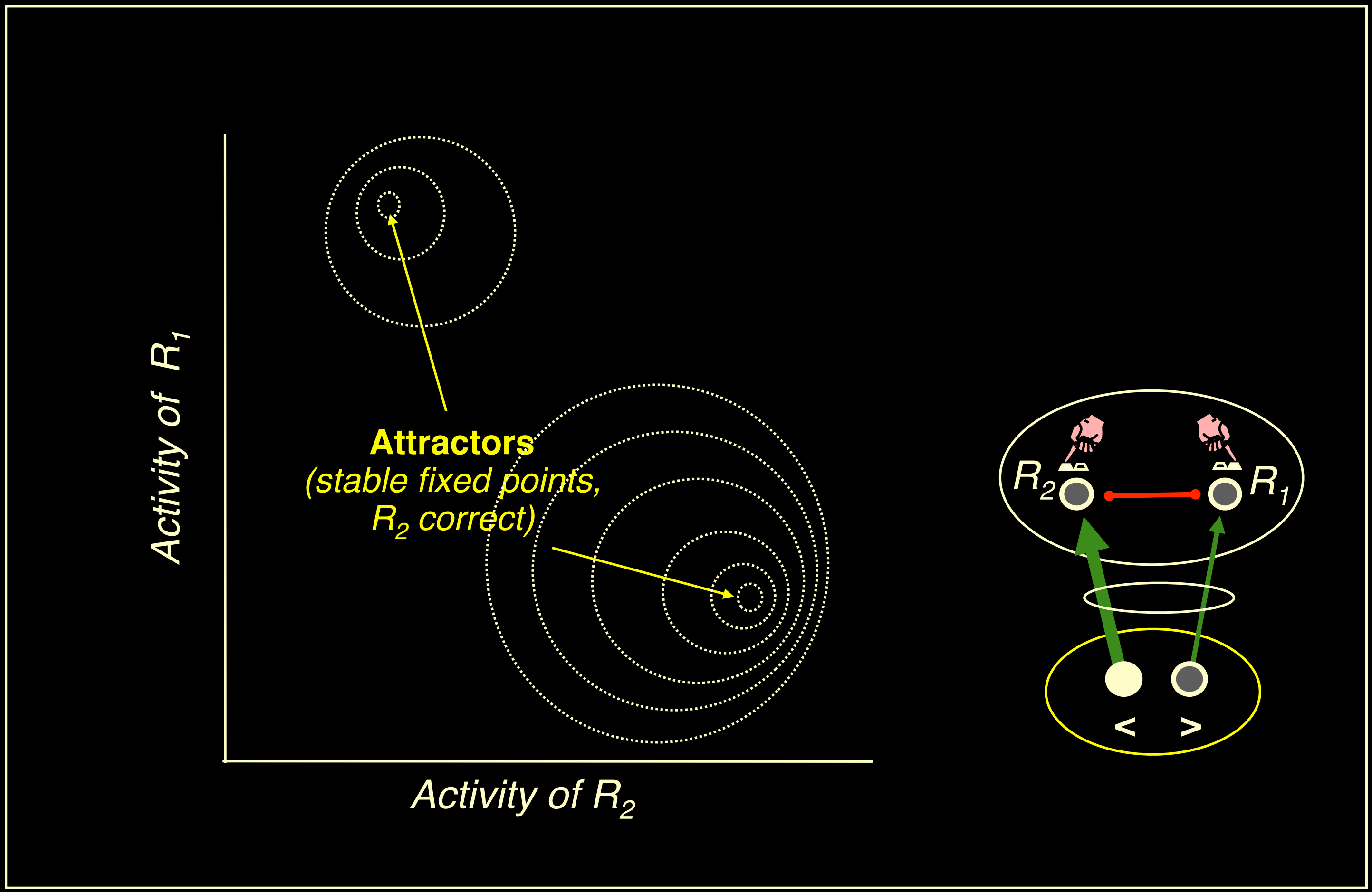

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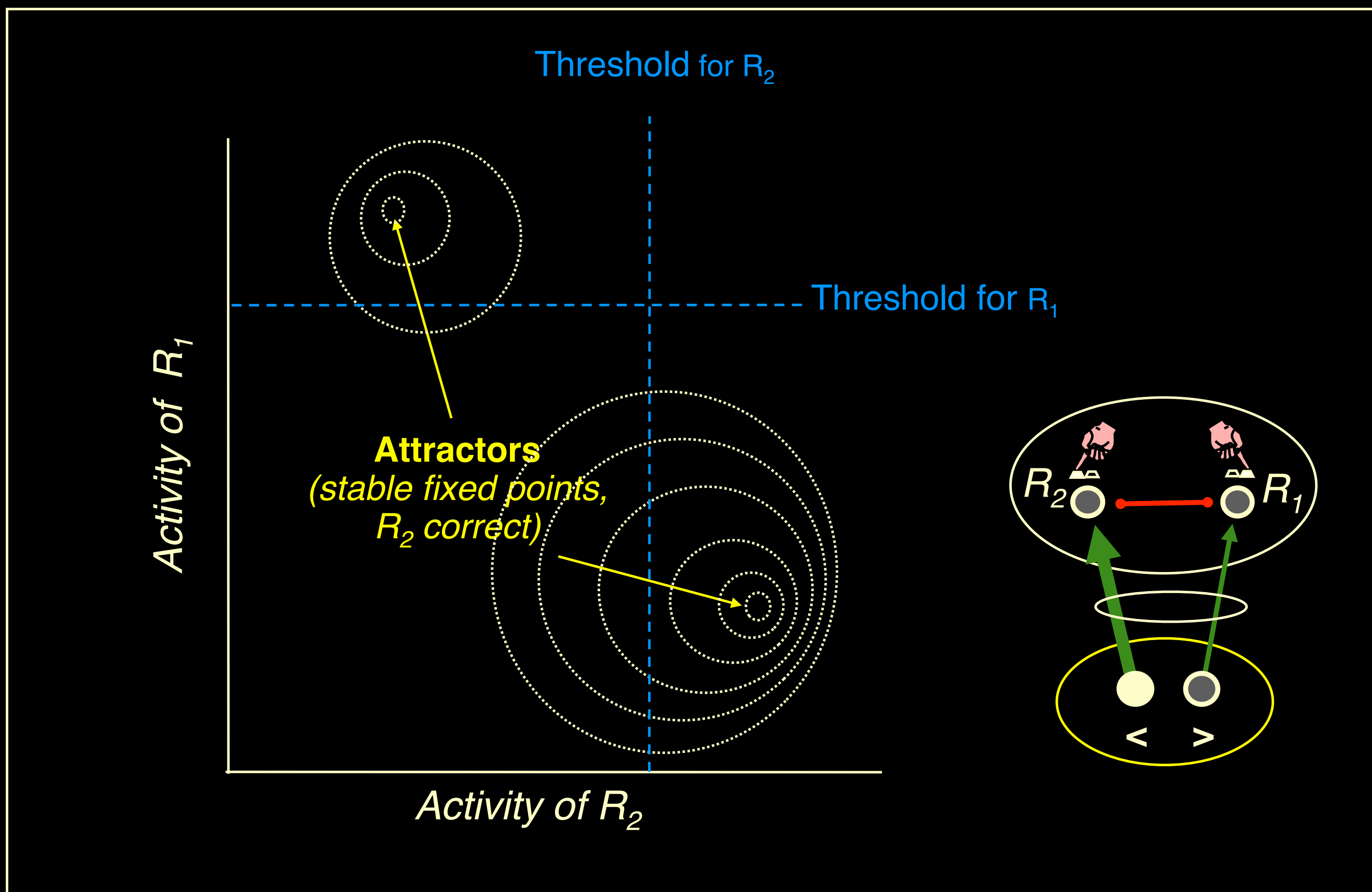

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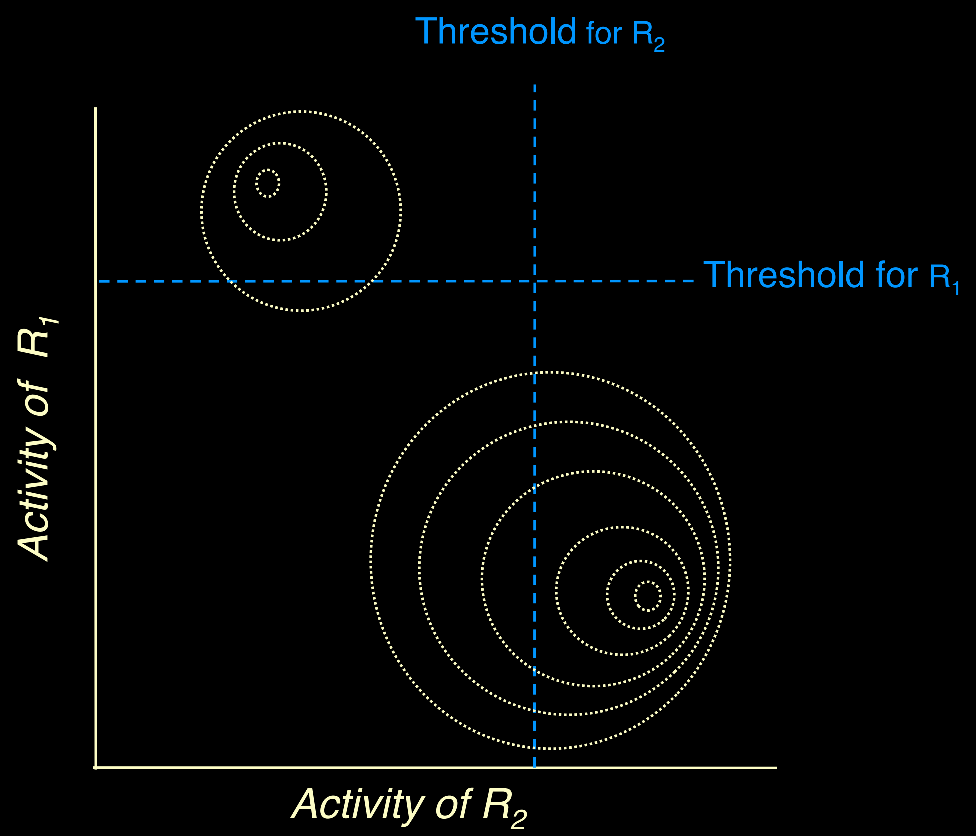

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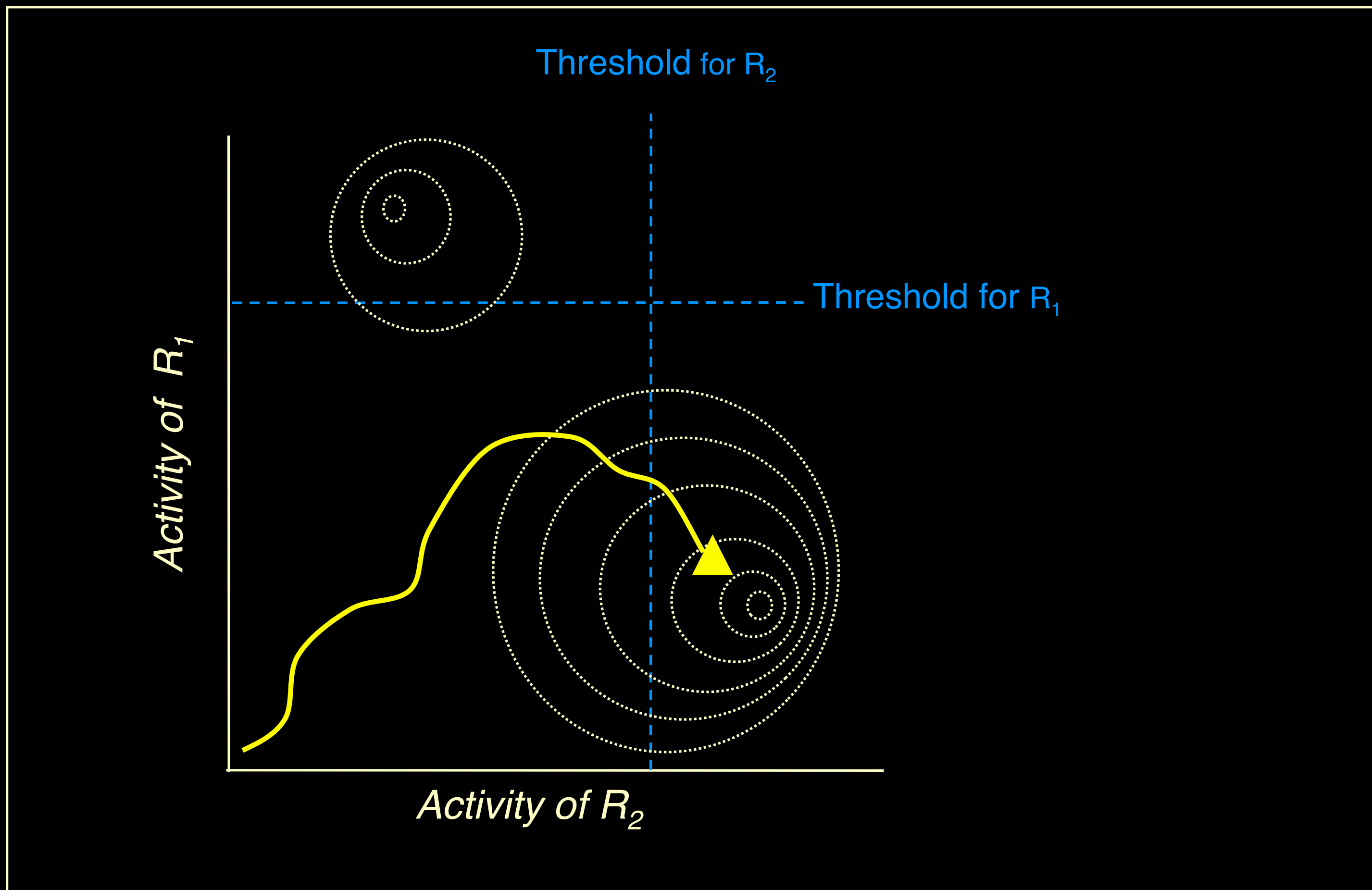

Simplification & Analysis

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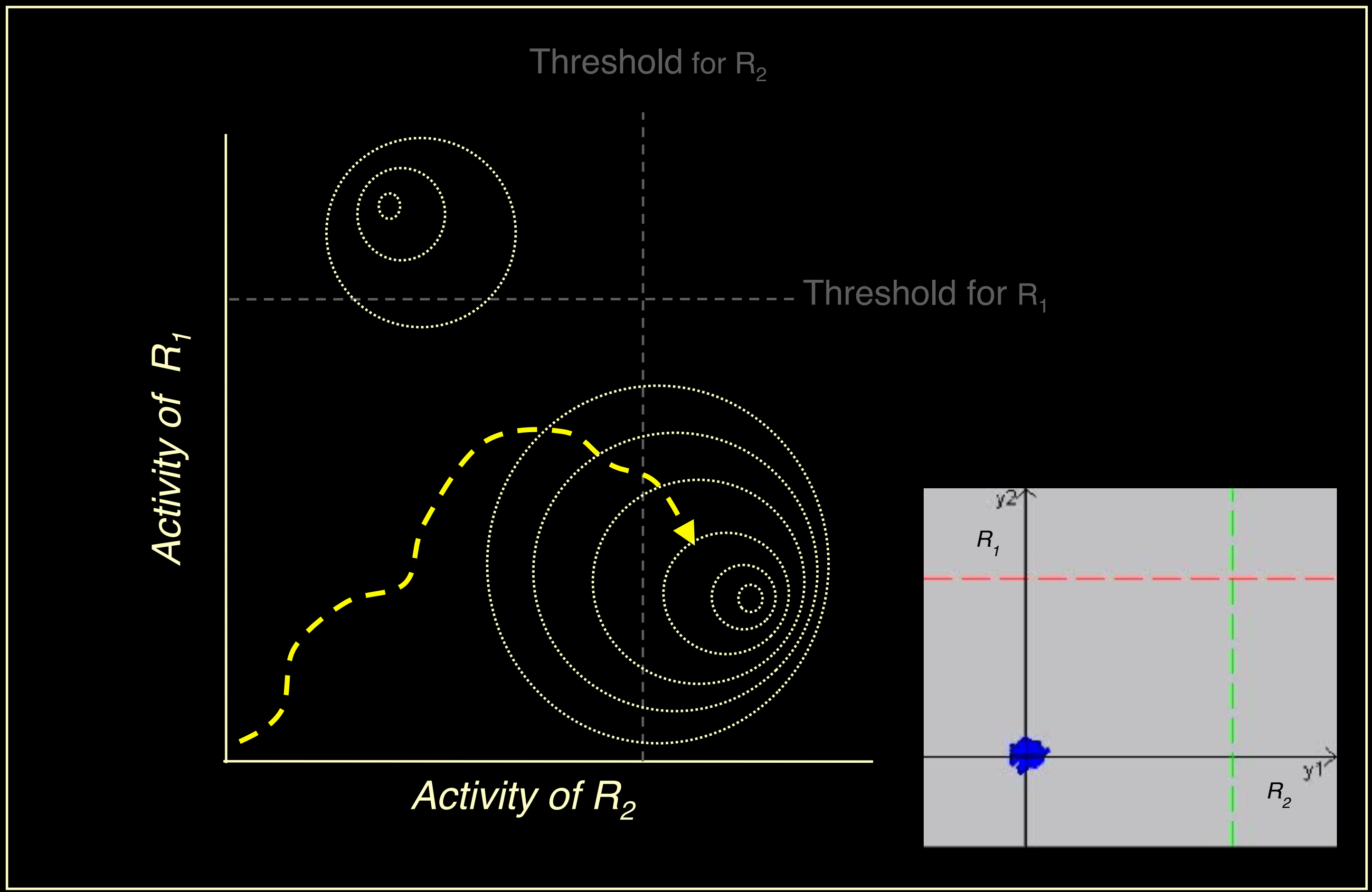

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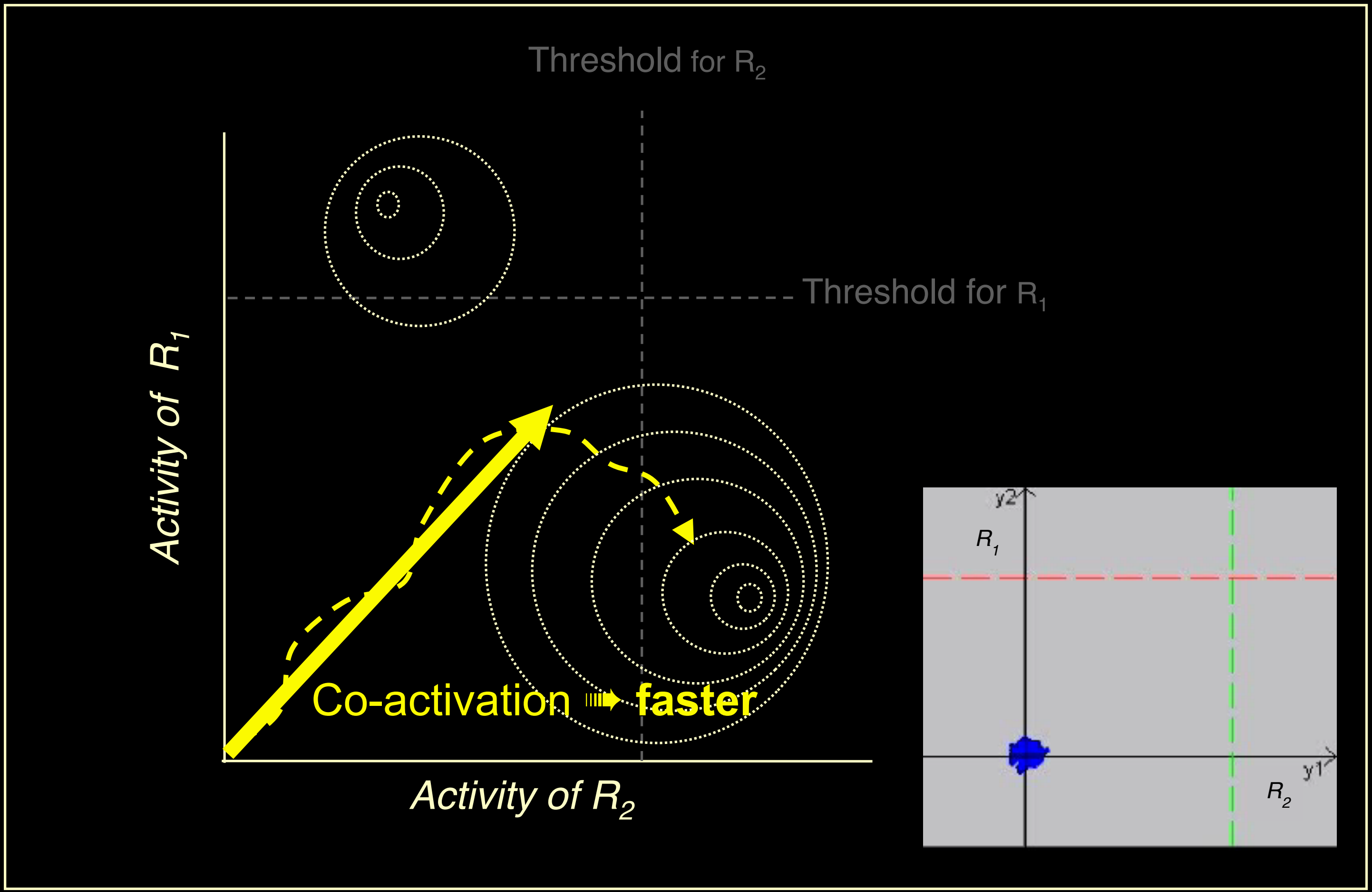

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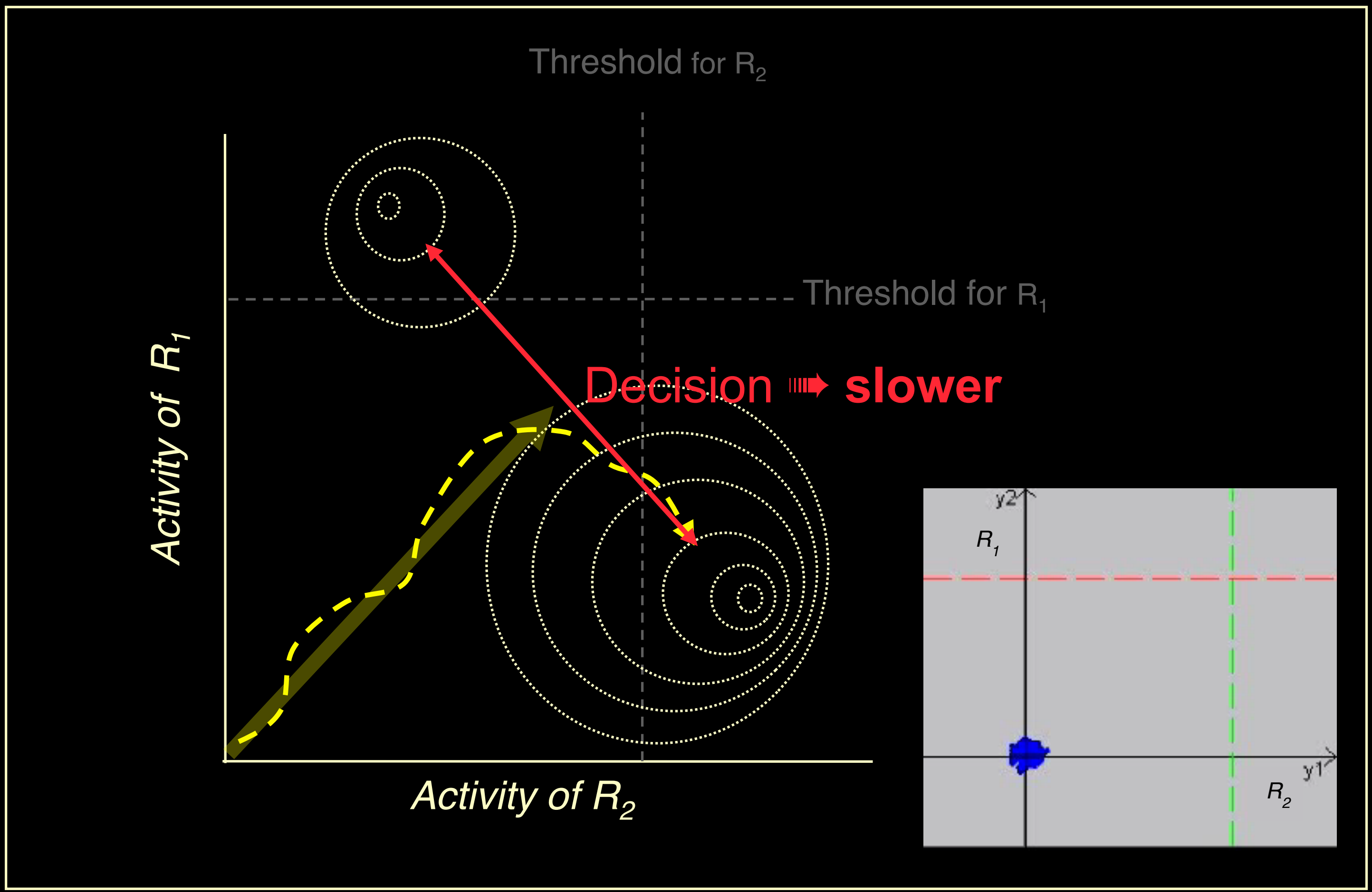

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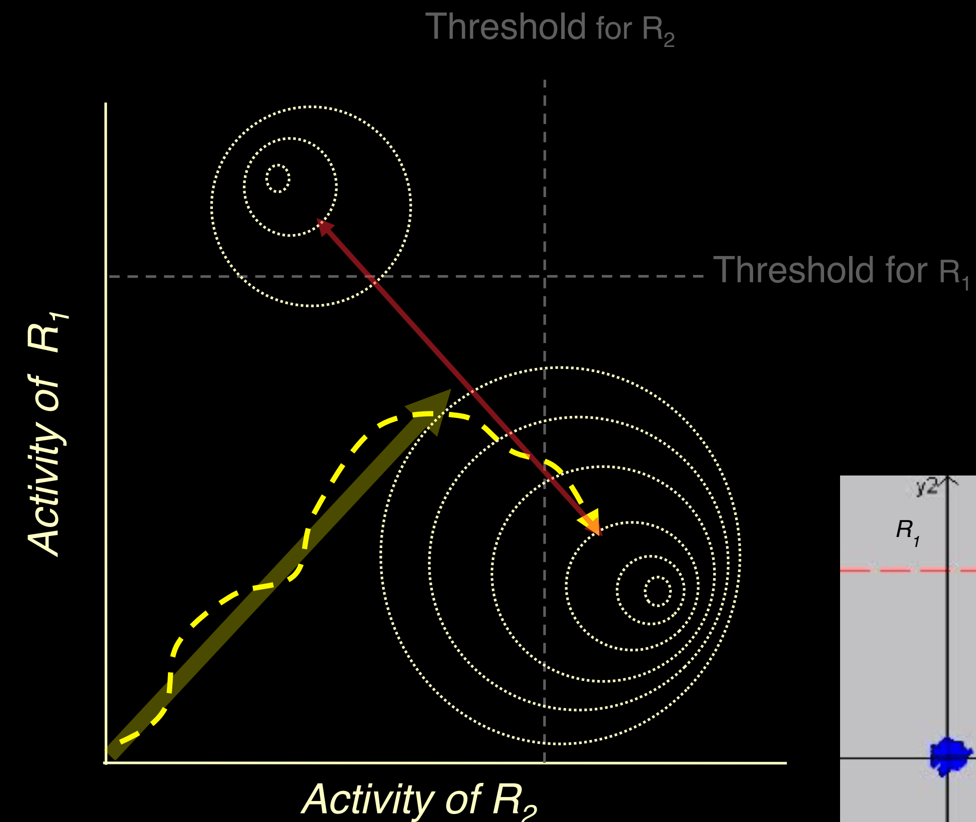

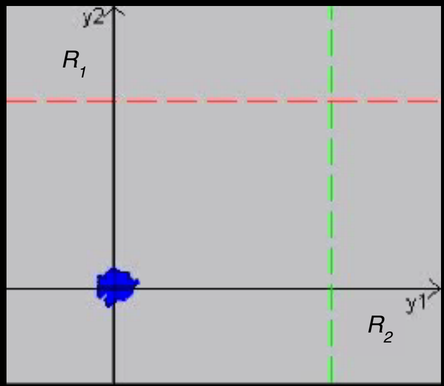

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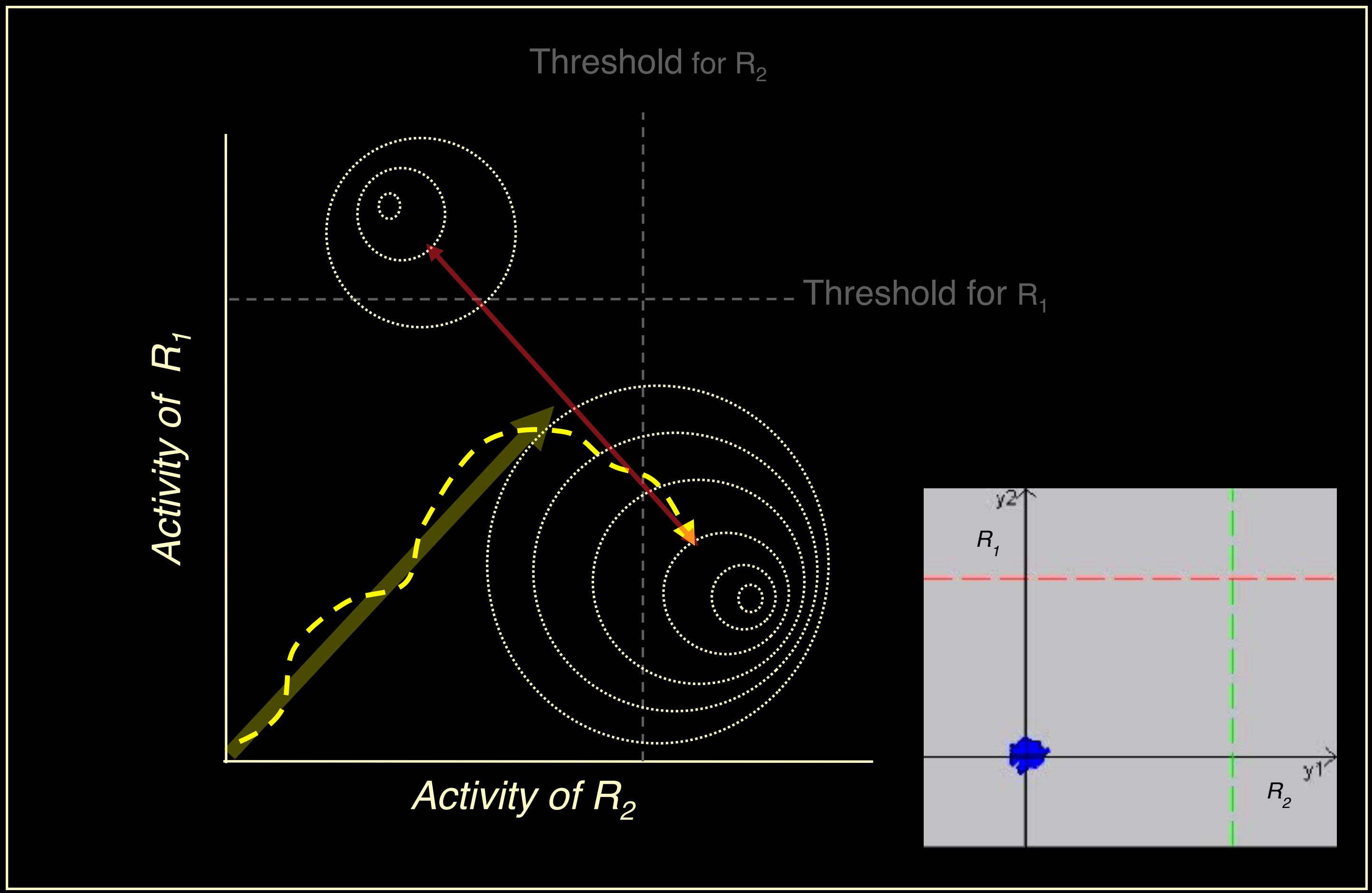

Simplification & Analysis

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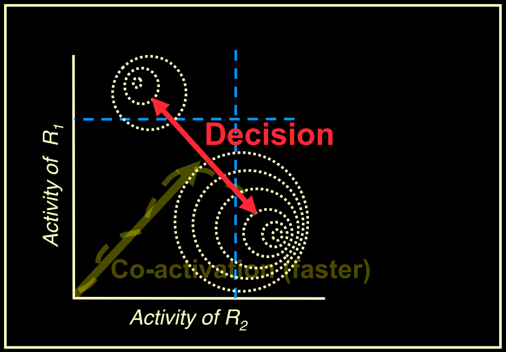

Simplification & Analysis

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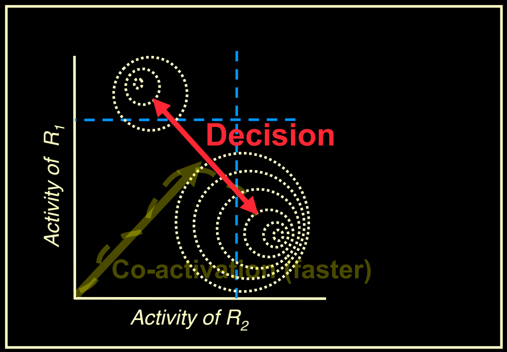

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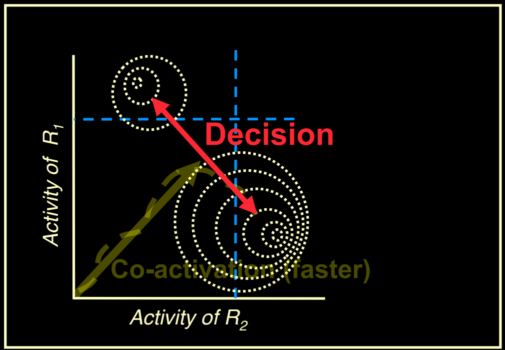

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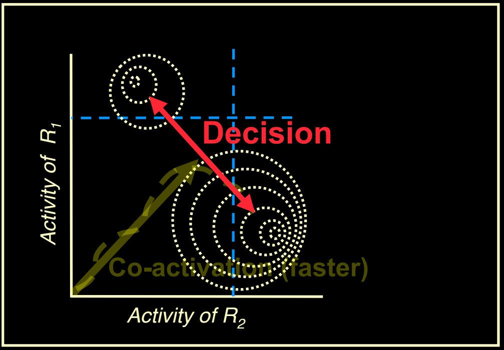

Simplification & Analysis

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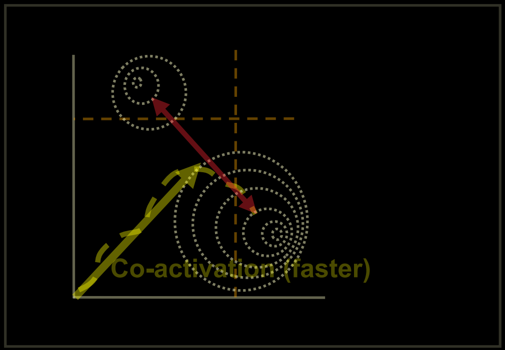

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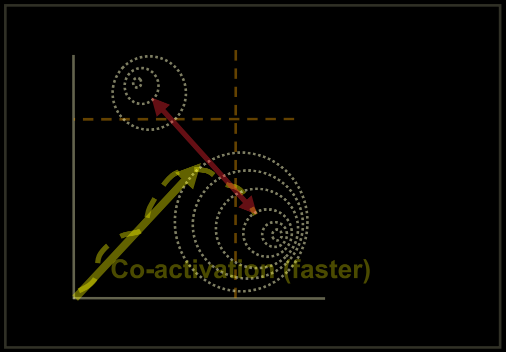
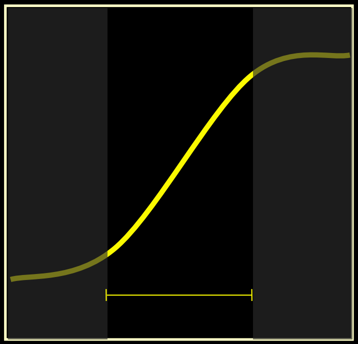
  therefore, decision process can be approximated by a *one-dimensional process* (*difference in activity*



- **Focus on linear range of function**



Simplification & Analysis

Step 5: Linearization

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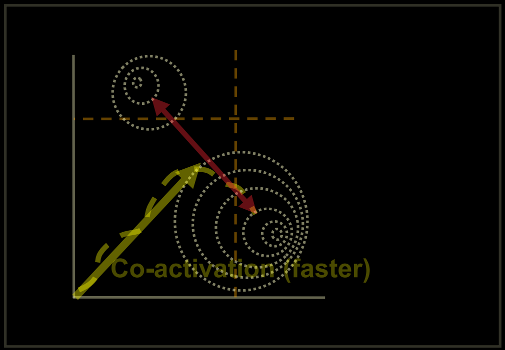
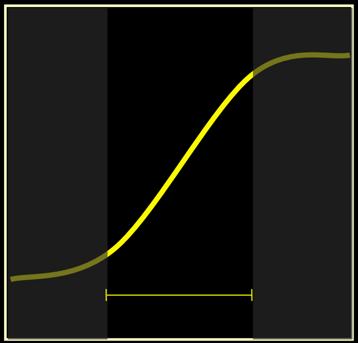


- **Focus on linear range of function**
  - **assume that units in the "focus of attention" are on the linear part of their activation function (i.e., most sensitive part of their dynamic range)**
    Cohen et al (Psychological Review, 1990)



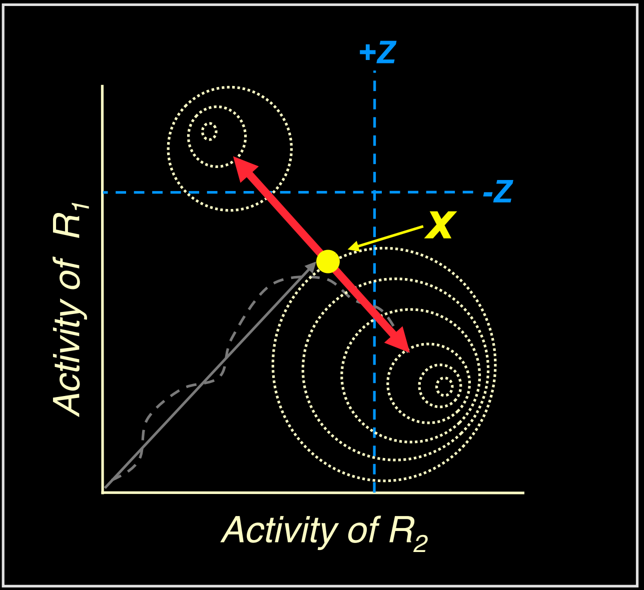
Simplification

- **Drift Diffusion Model (DDM):**

$$\dot{x} = A + c$$

A = drift rate
c = noise
P(x,t) = N(At, c√t)

Process ends when x exceeds ± z



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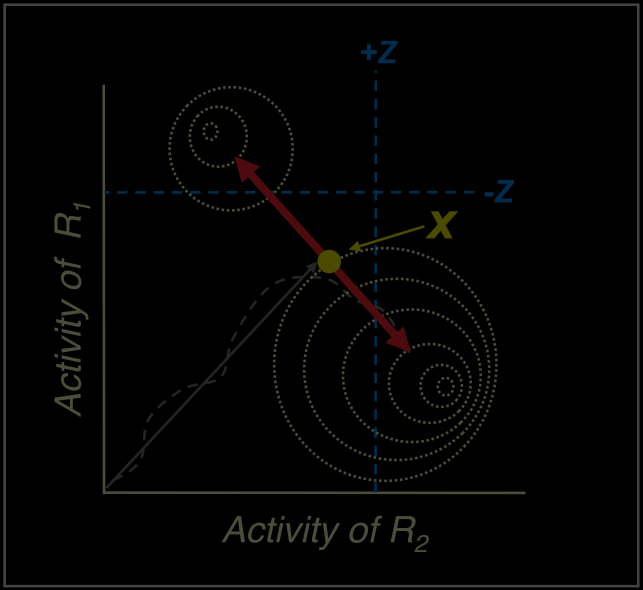

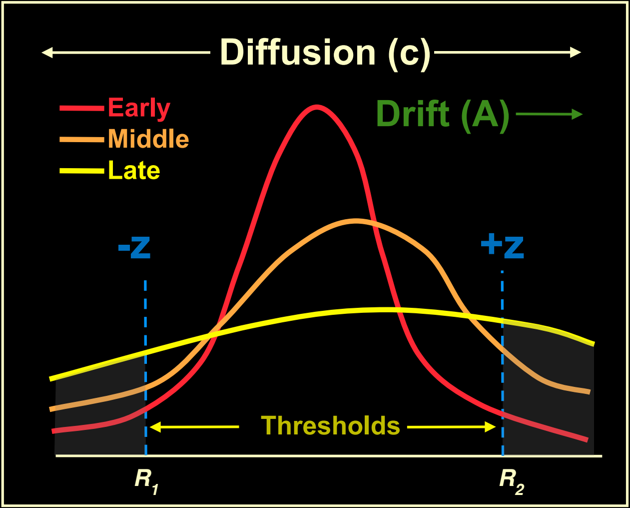

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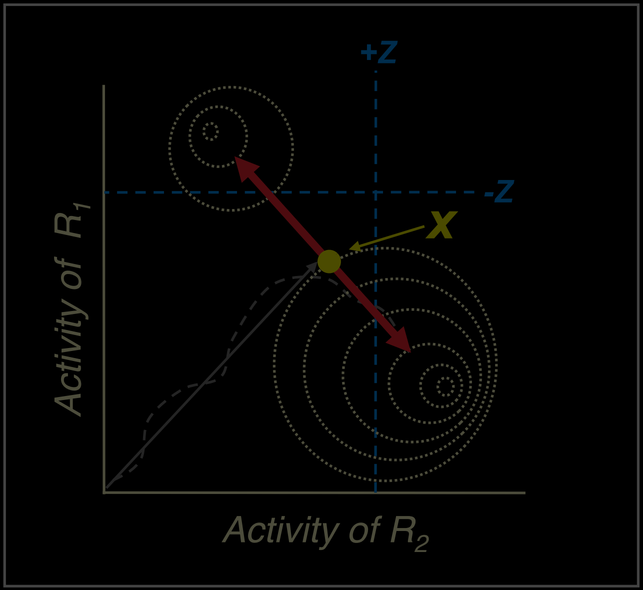

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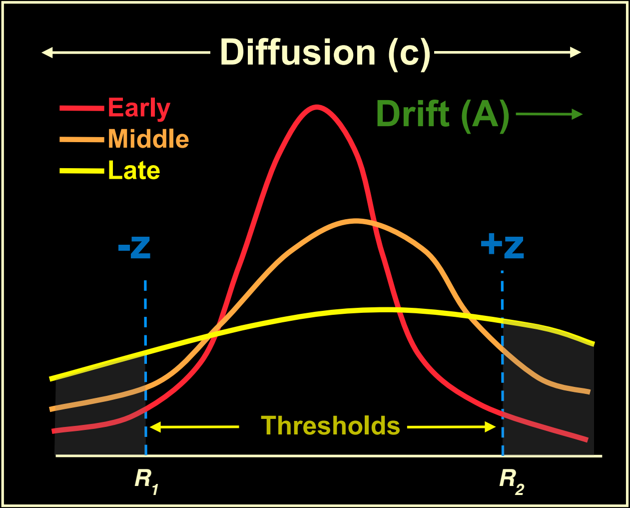
  = mathematical description of diffusion of an ideal gas



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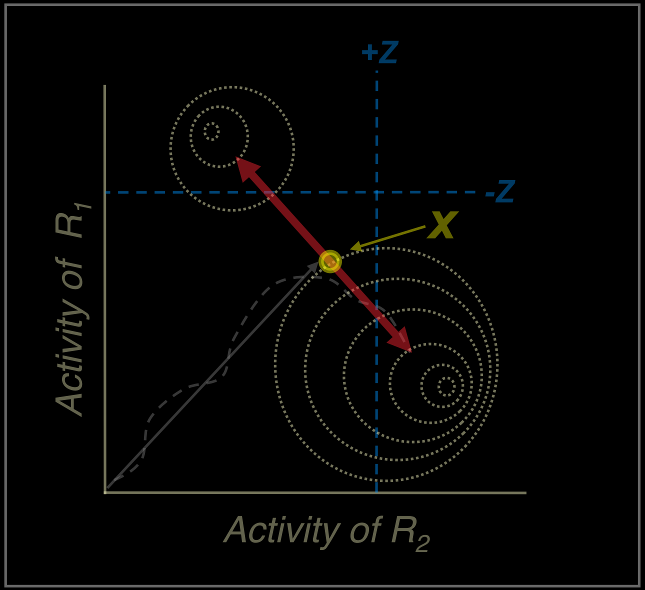

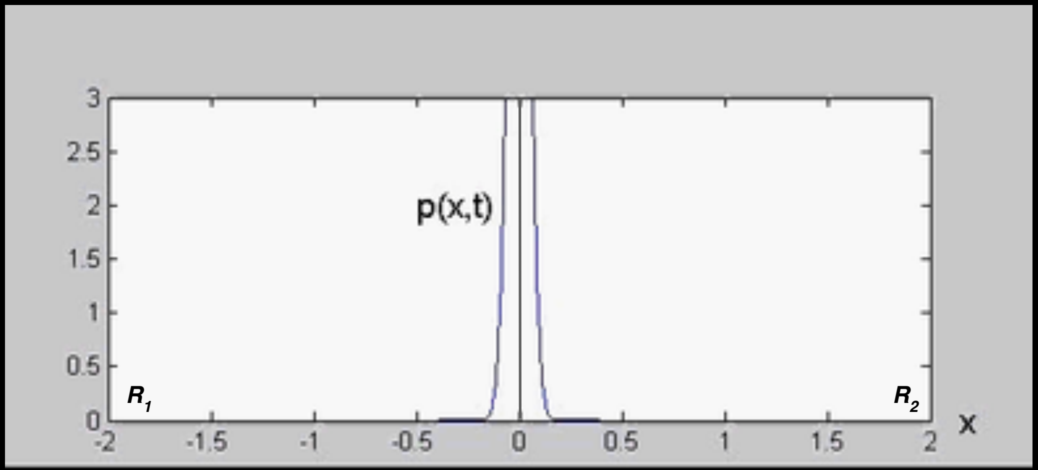

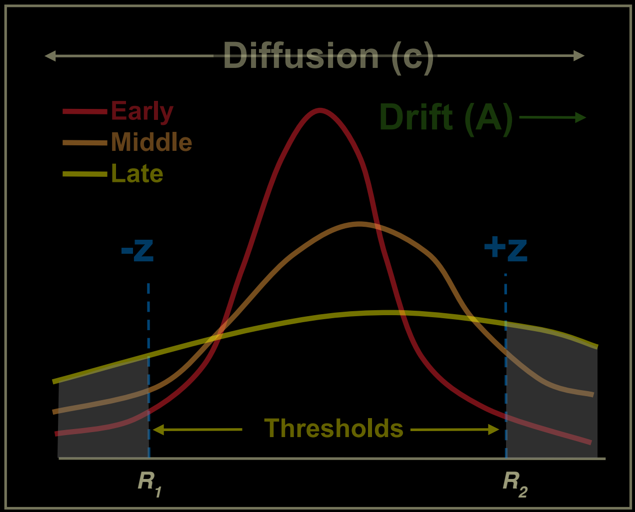

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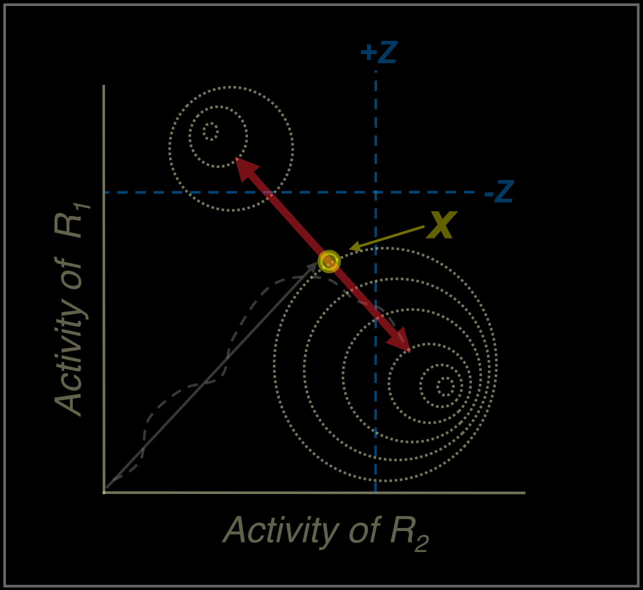

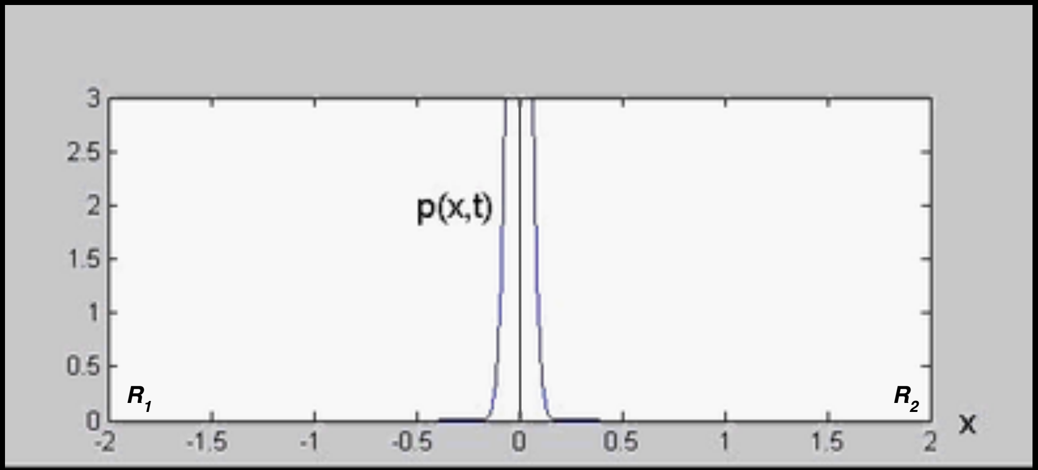

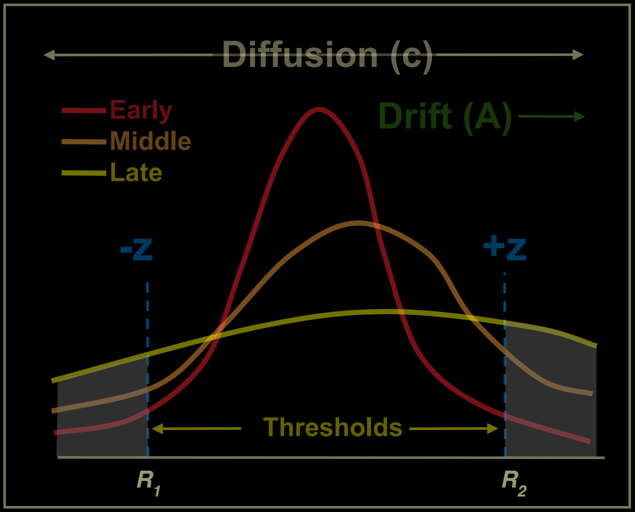

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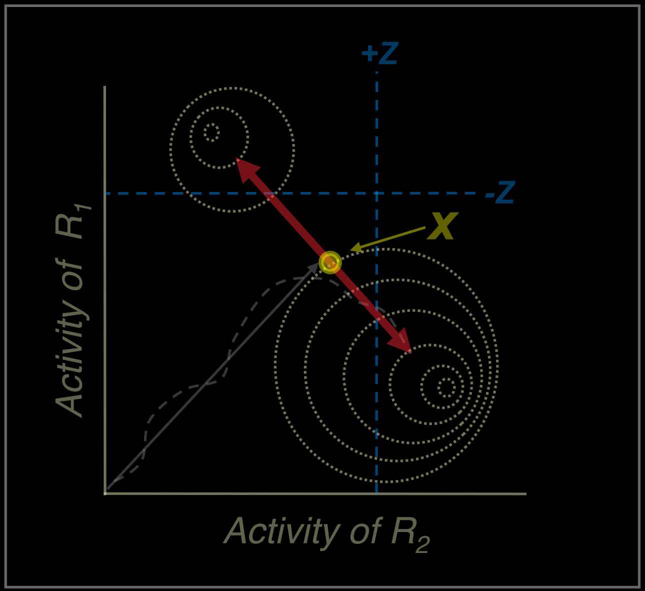

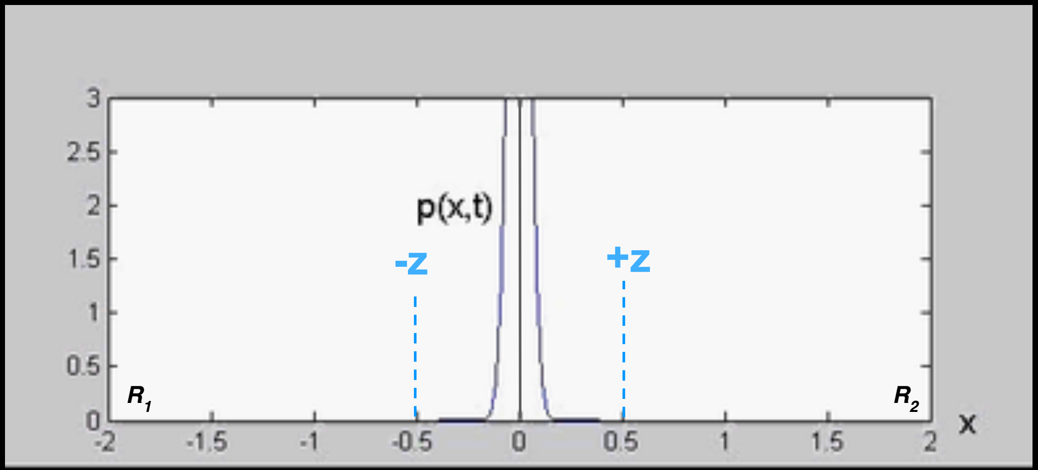

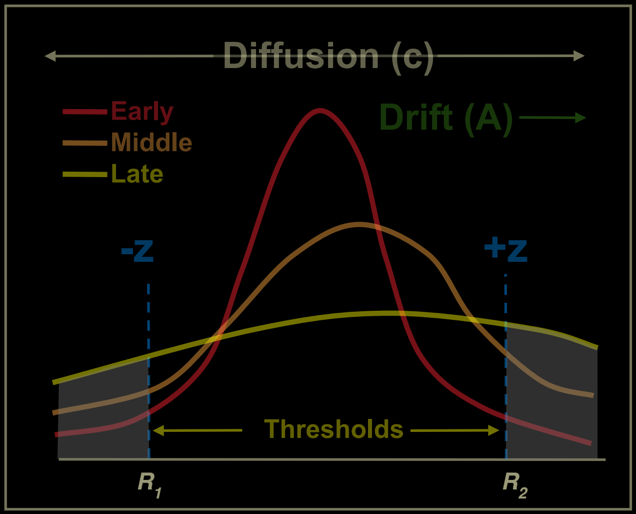

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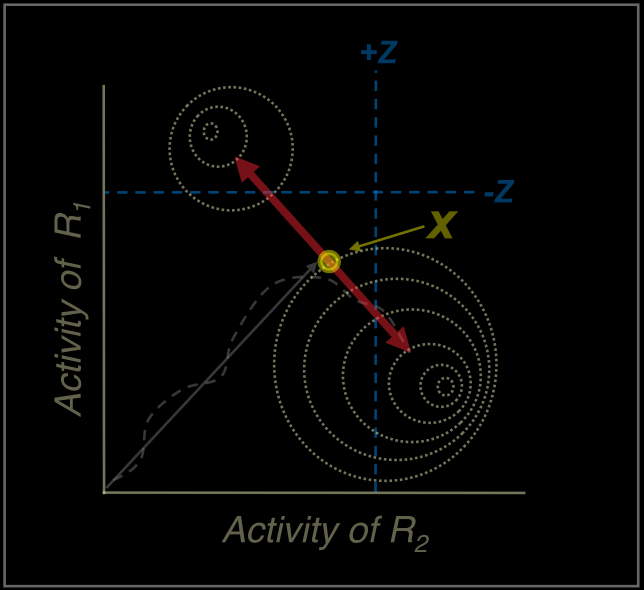
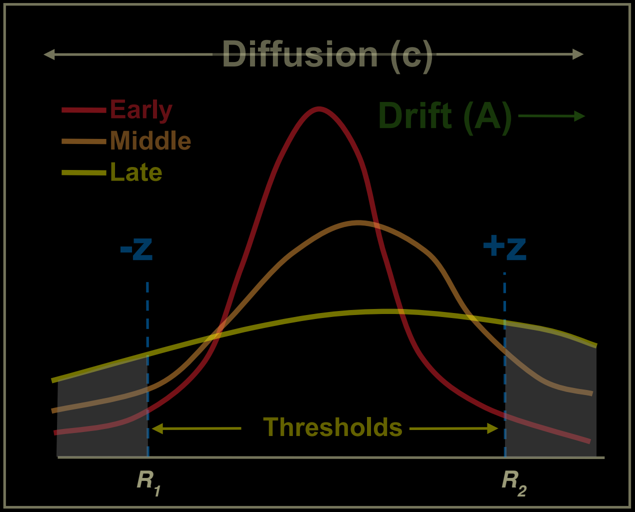


- **Can analytically solve for Error Rate and Decision Time:**

$$\text{Error Rate (ER)} = \frac{1}{1 + e^{\frac{2 \cdot \text{Drift} \cdot \text{Threshold}}{\text{Noise}^2}}}$$

$$\text{Decision Time (DT)} = \frac{\text{Threshold}}{\text{Drift}} \tanh\left(\frac{\text{Drift} \cdot \text{Threshold}}{\text{Noise}^2}\right)$$



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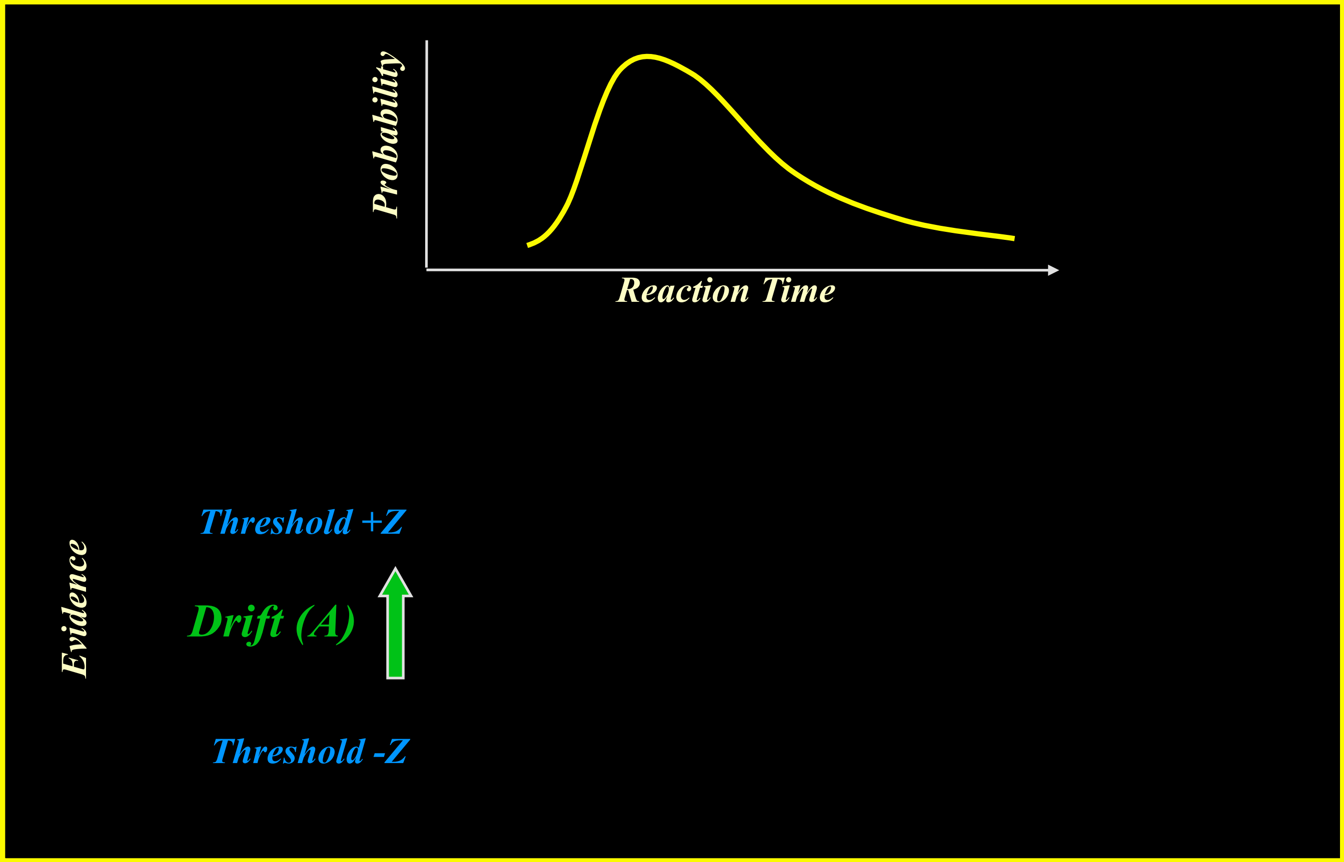

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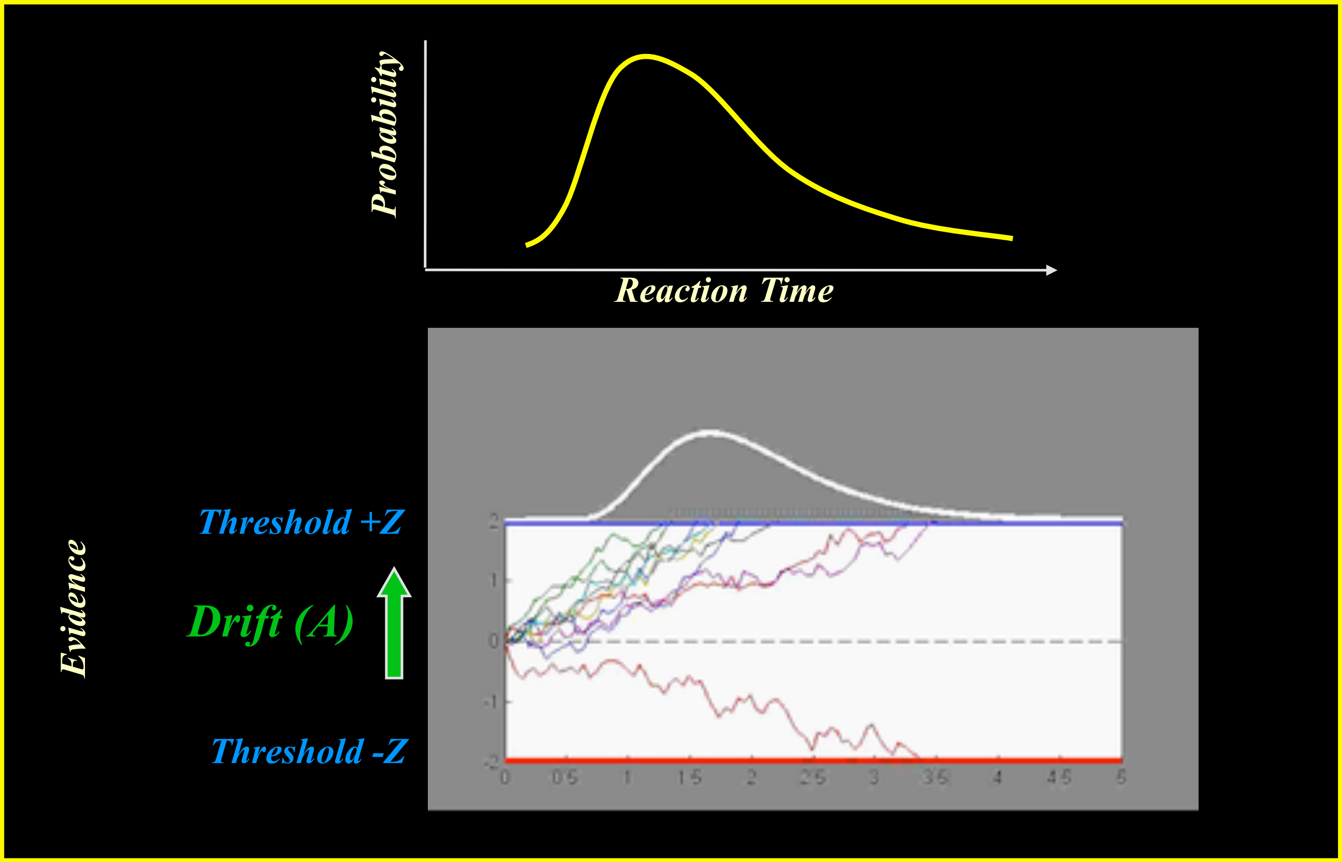

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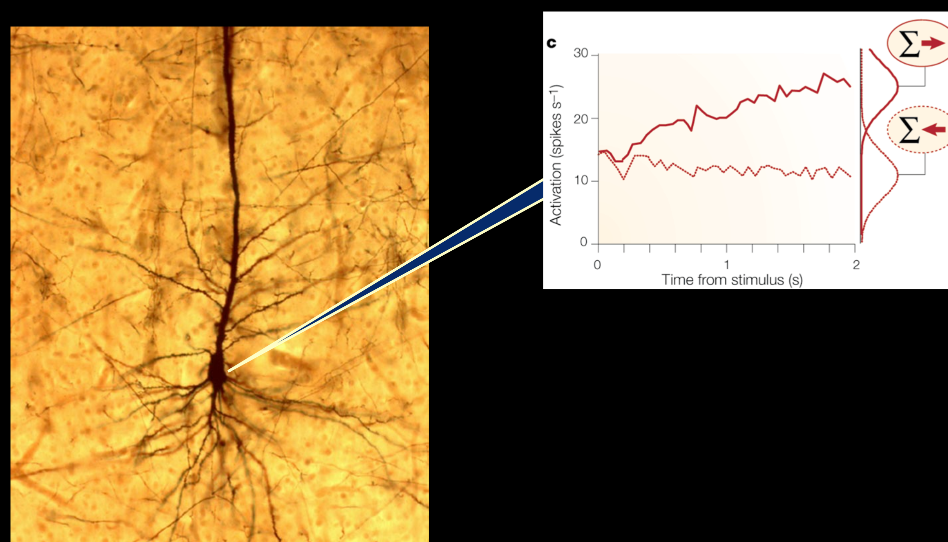

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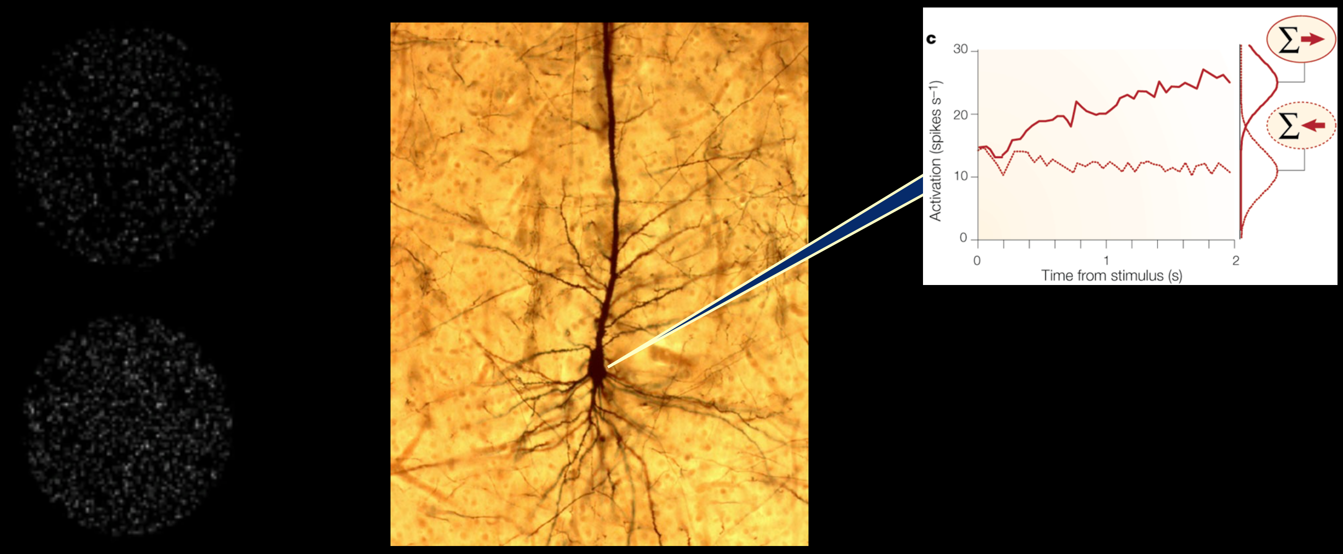

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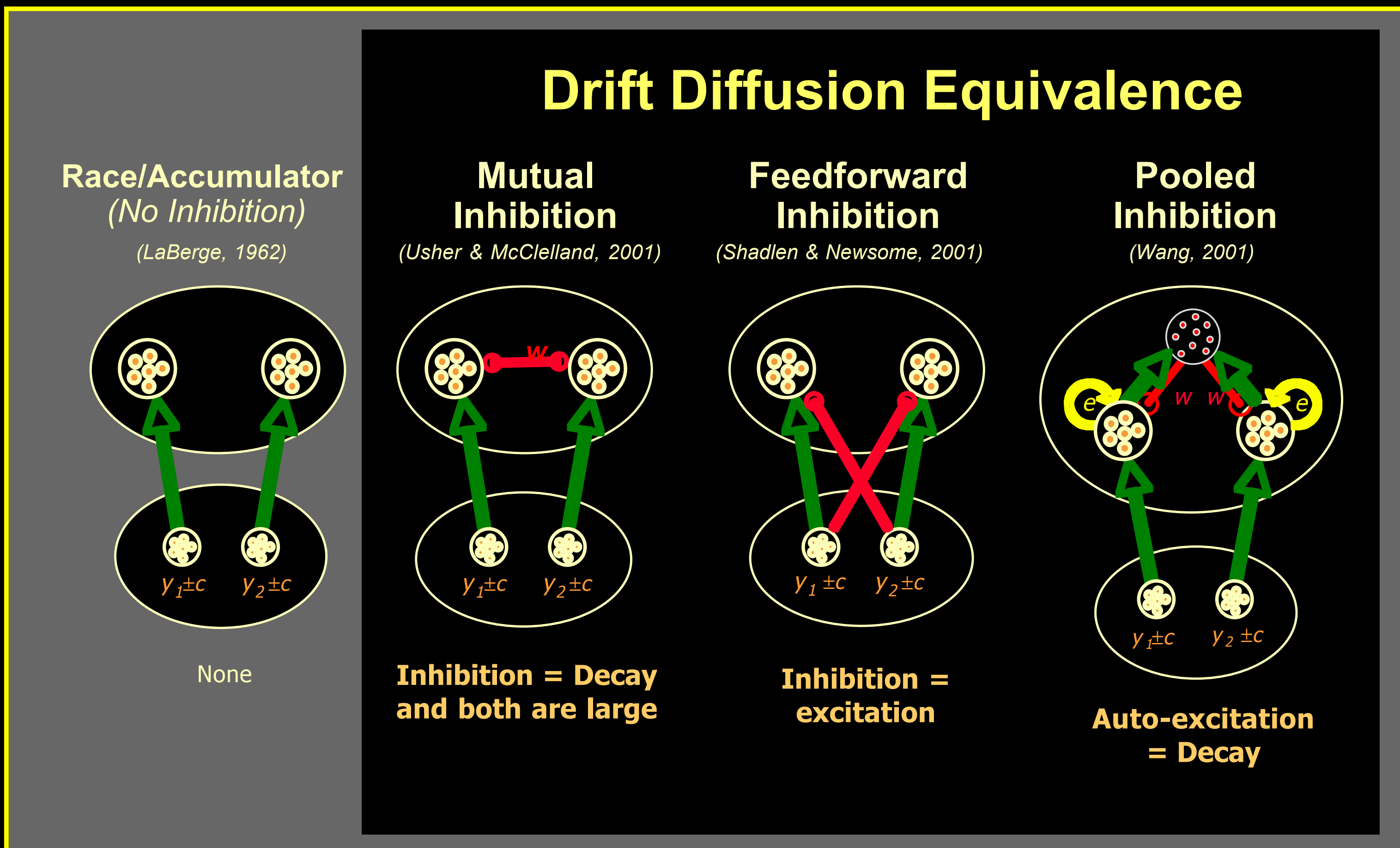

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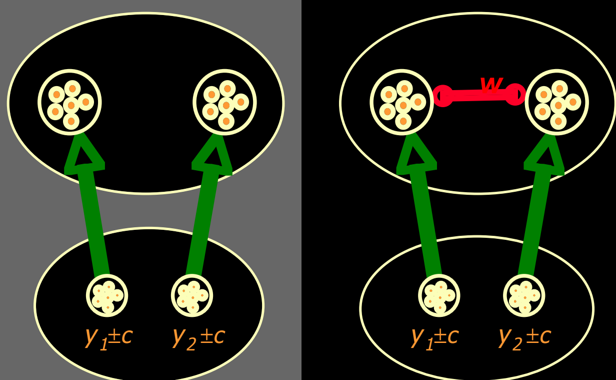
Drift Diffusion Equivalence

Race/Accumulator
(No Inhibition)
(LaBerge, 1962)

Mutual Inhibition
(Usher & McClelland, 2001)



None

Inhibition = Decay and both are large

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  - **Here is where** ***control*** **comes in…**

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  - First, however, must define *"objective function"*
    → the function that control seeks to optimize