Optimization and Control of Decision Making

Theoretical Traction

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- **Optimal decision making process**
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  - **Can think of this in terms of optimization** ***control*** **comes in…**

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    (for example, what is the optimal threshold, and do people use this?)
  - First, however, must define *"objective function"*
    → the function that control seeks to optimize

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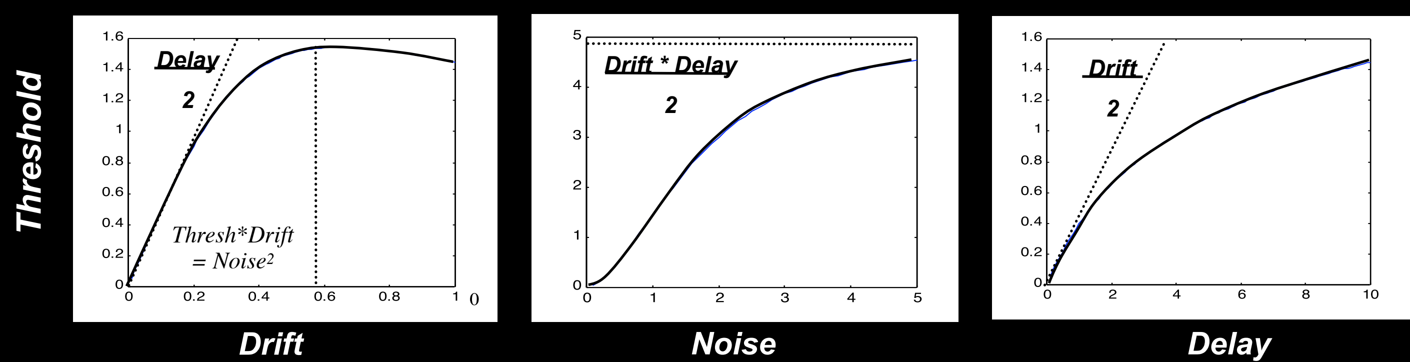

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Reward Rate Optimization

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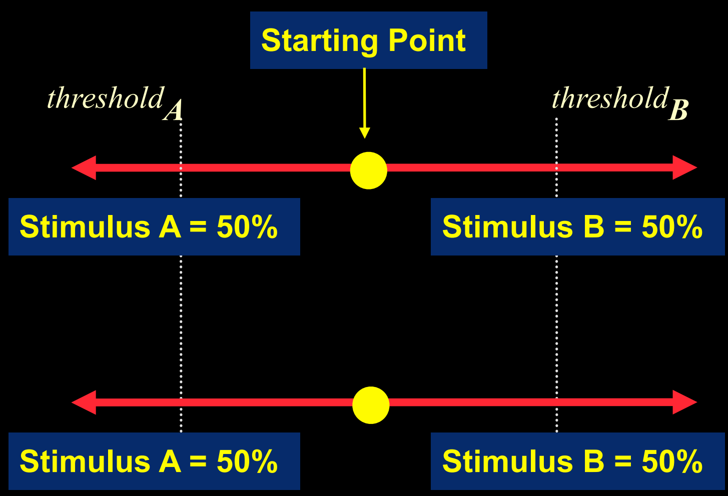
- Re-express RT and ER in terms of DDM parameters:
- Solve for threshold that maximizes RR:
- Predict changes in speed-accuracy tradeoff (threshold) that *optimize RR* as a function of task parameters: delay, drift, and noise

Predictions: Effects of Stimulus Frequency

- If one stimulus is more frequent than the other:
  - it is optimal to move the starting point (“prior”), *not* the threshold *(assuming constant drift [SNR]):*



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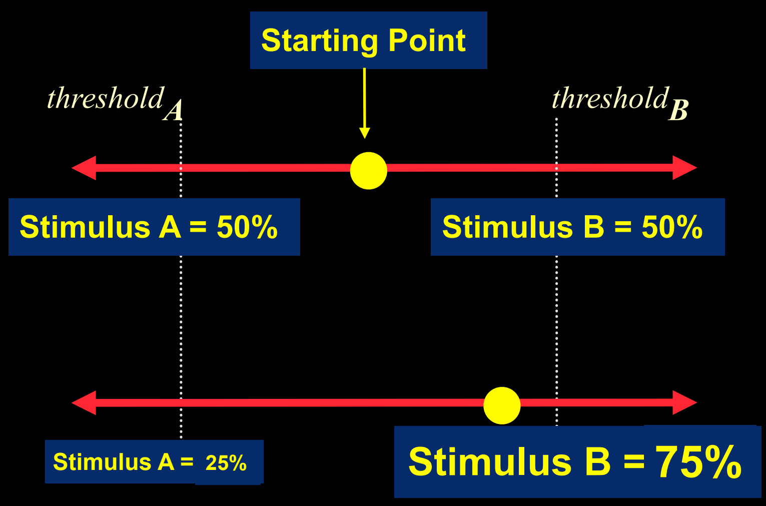

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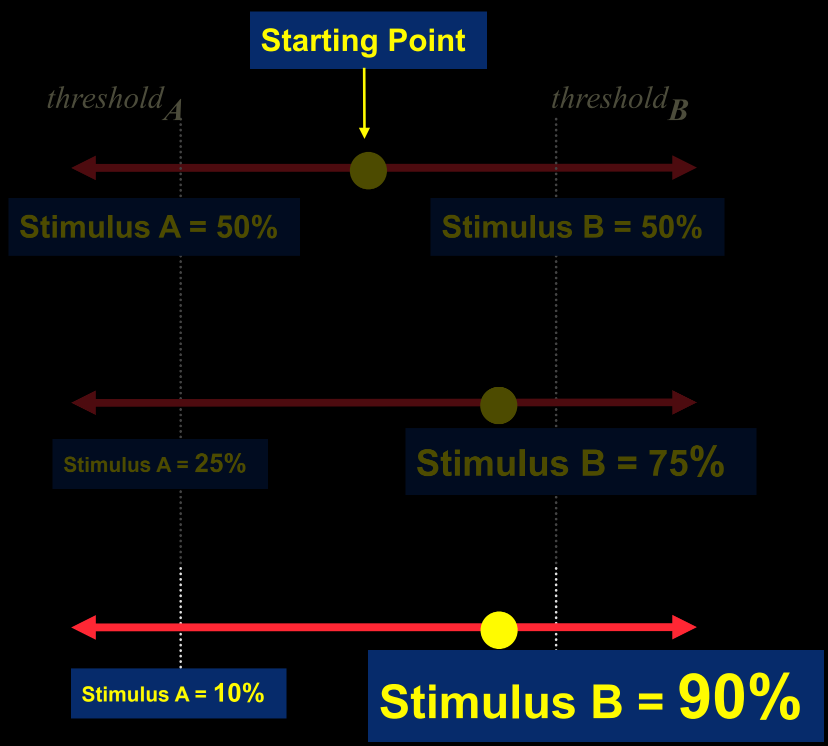

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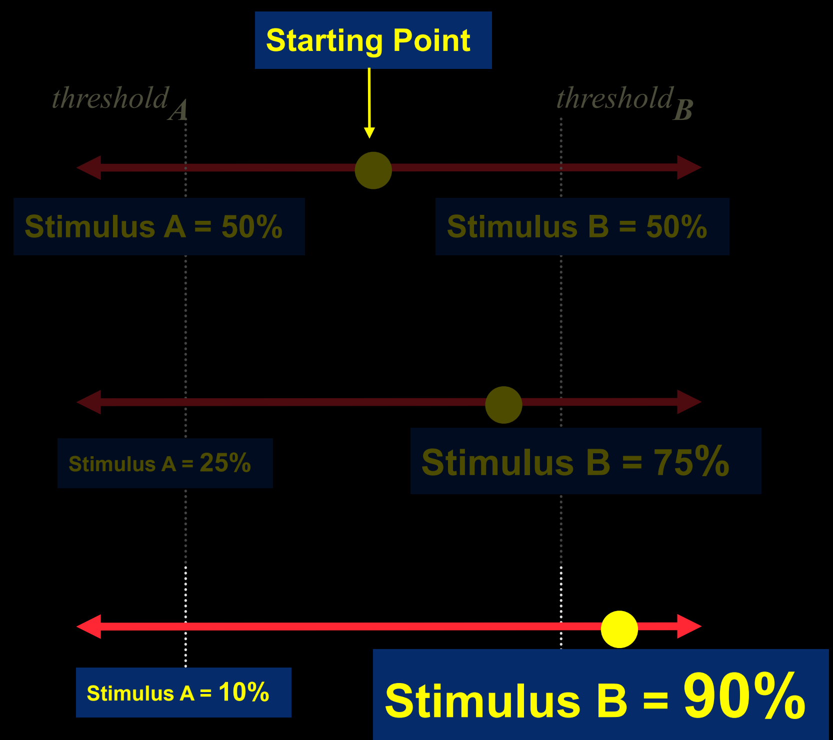

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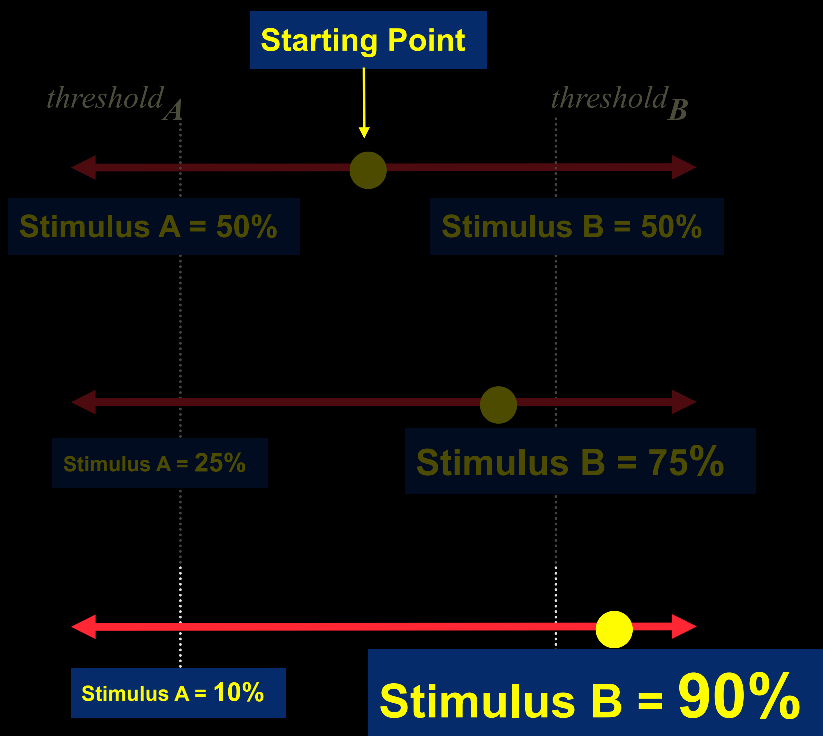

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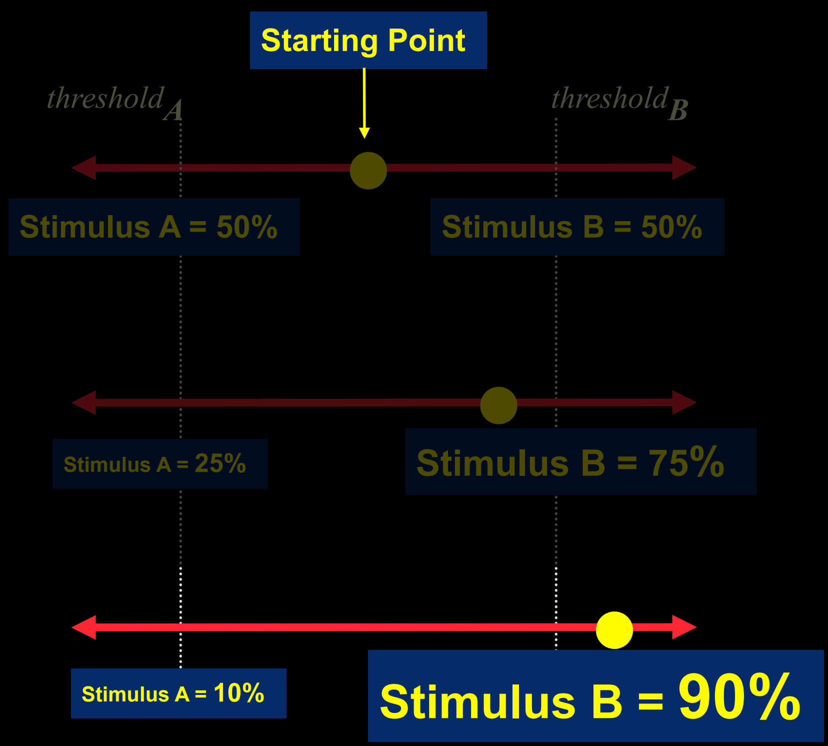

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- **For sufficiently extreme frequencies:**
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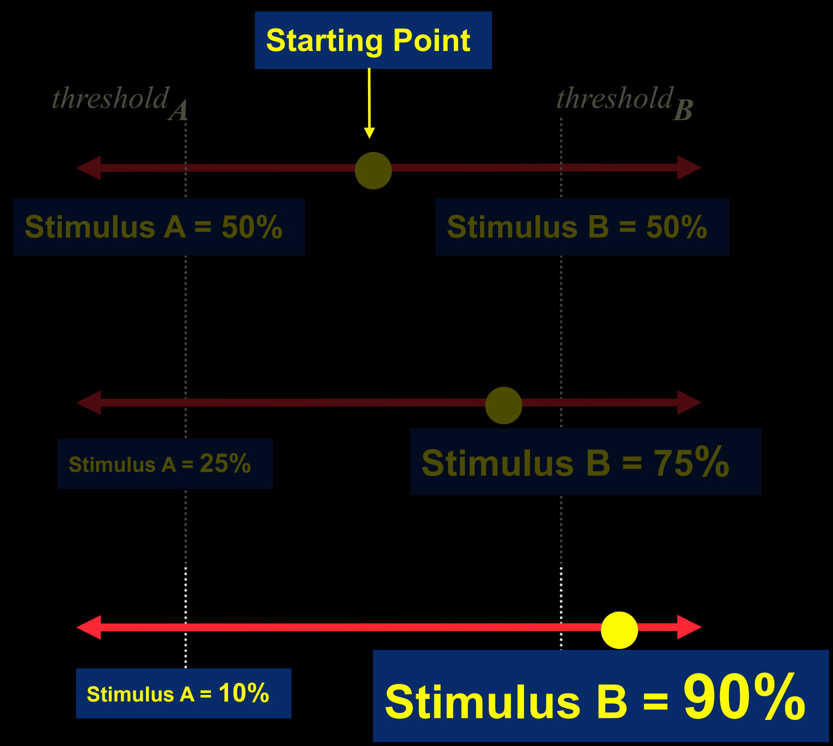
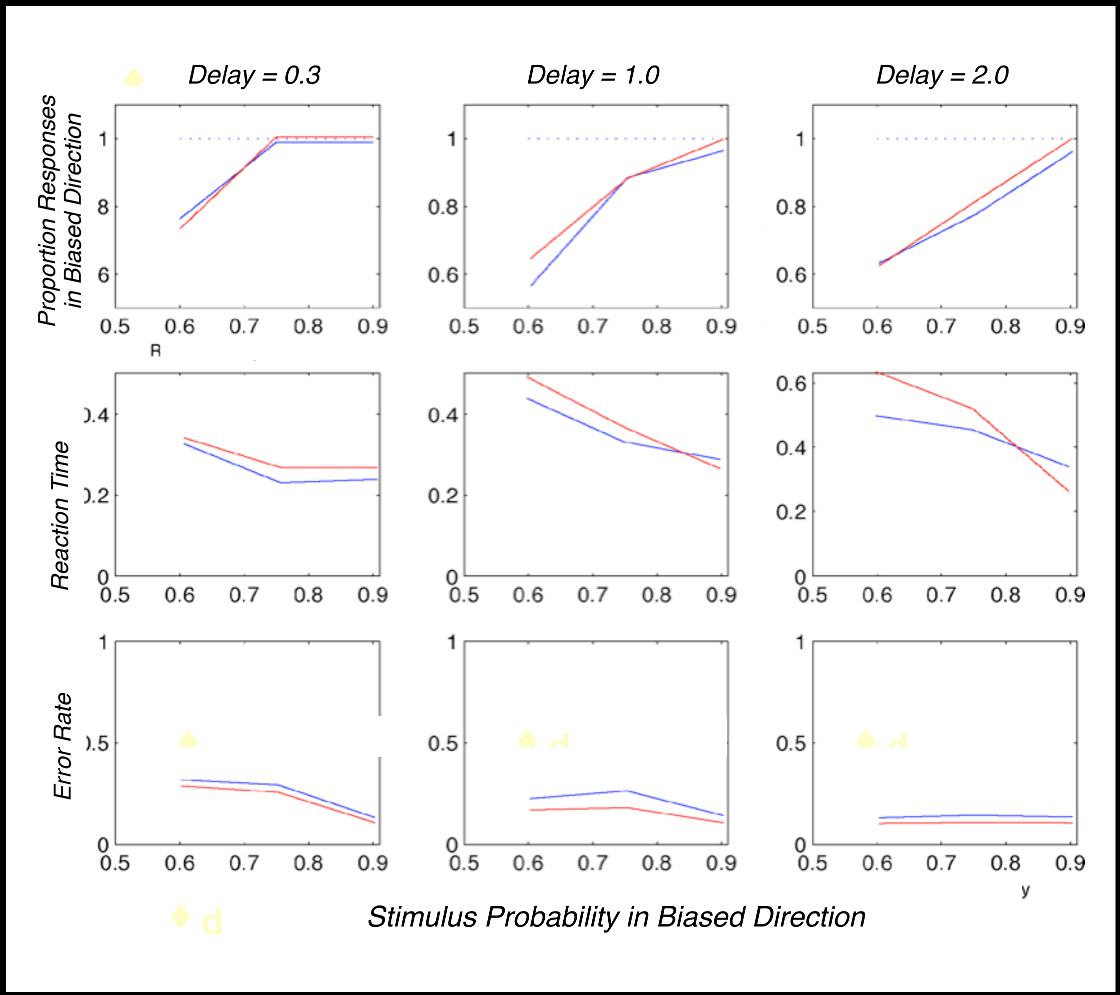
  - **the stimulus frequency at which this occurs varies according to delay and drift…**



Data: Effects of Stimulus Frequency

Empirical Data

(Simen et al., 2009)



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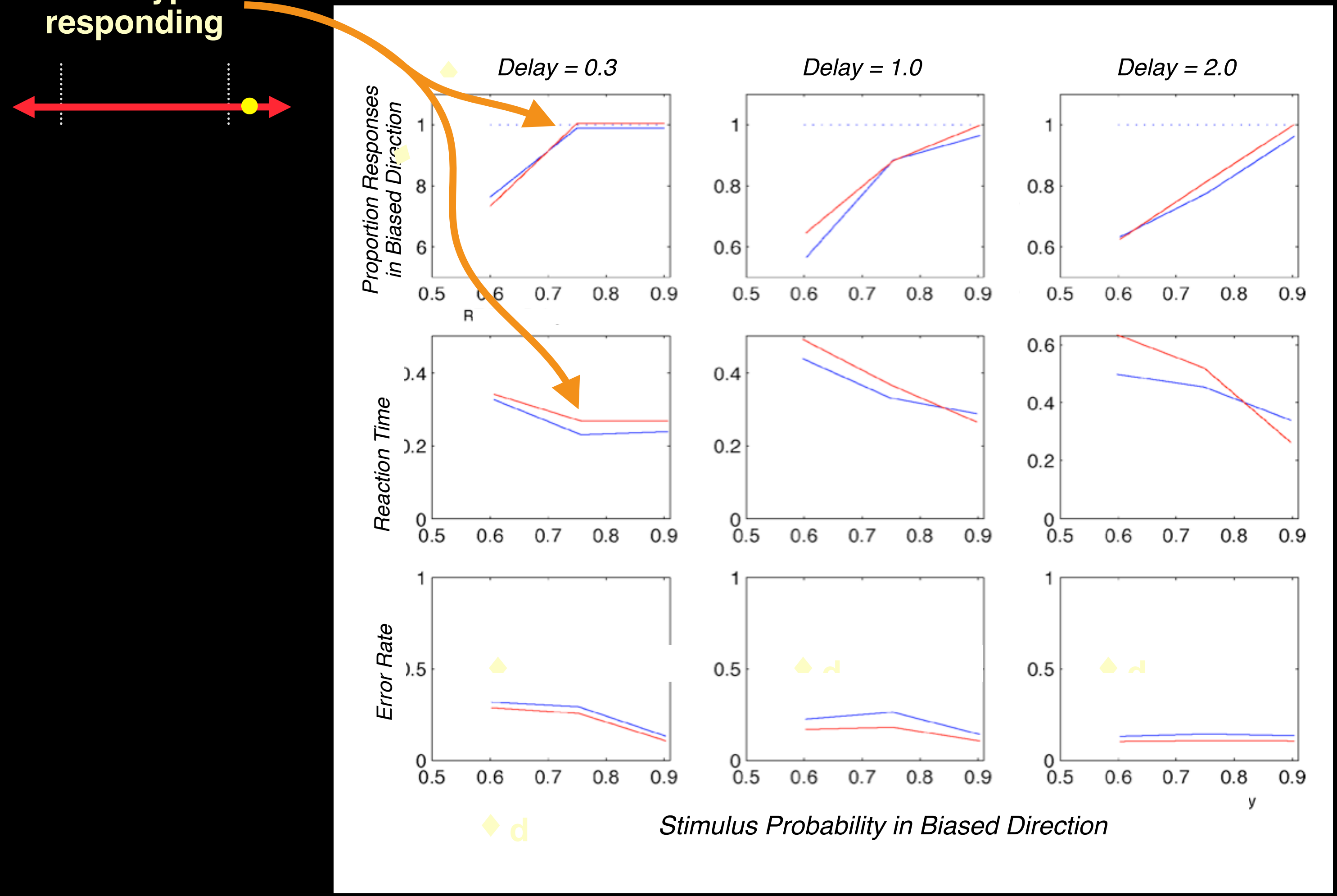

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- In other words, there is a *single optimal speed-accuracy curve* that should quantitatively define performance…

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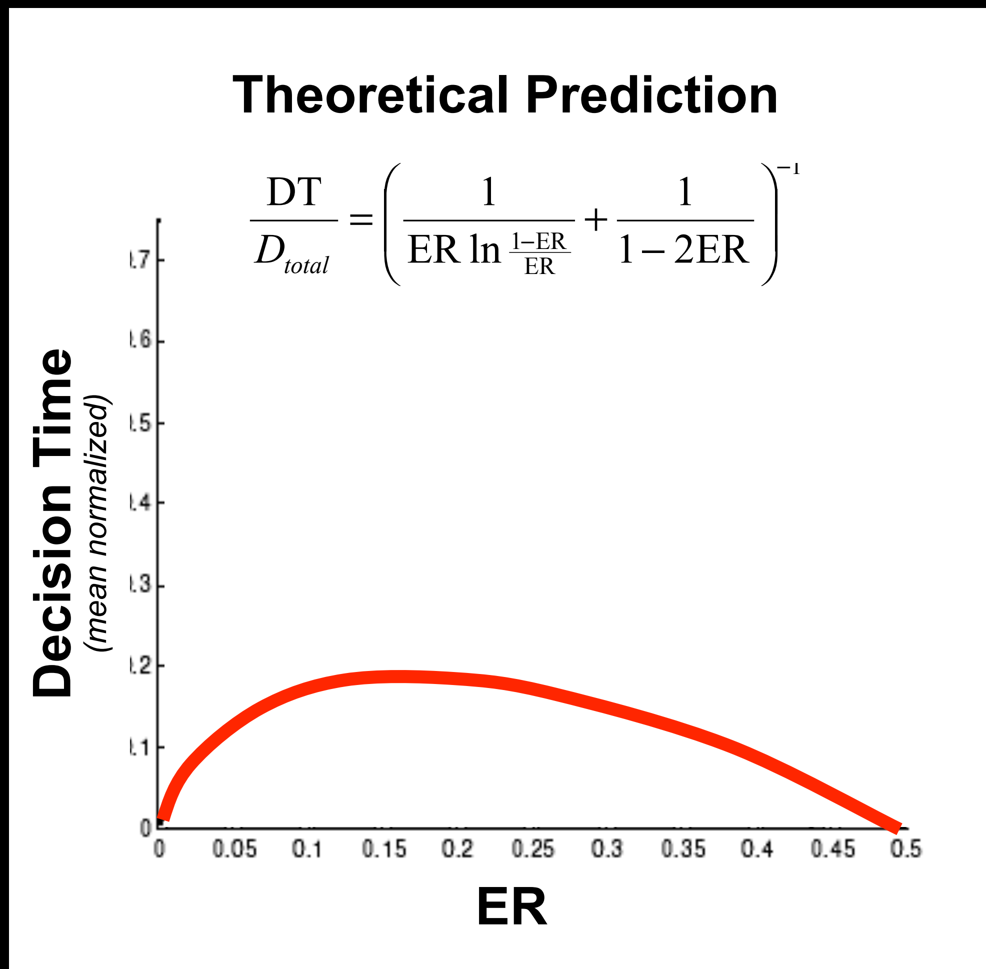

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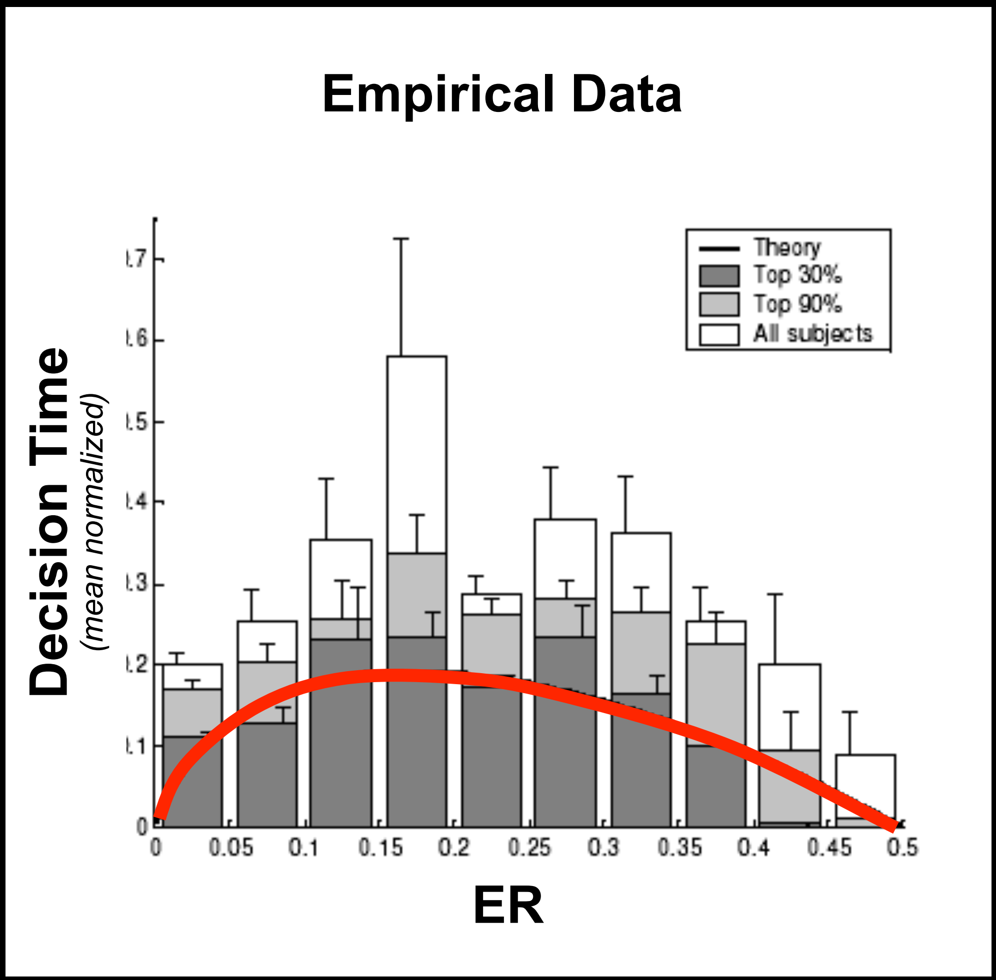

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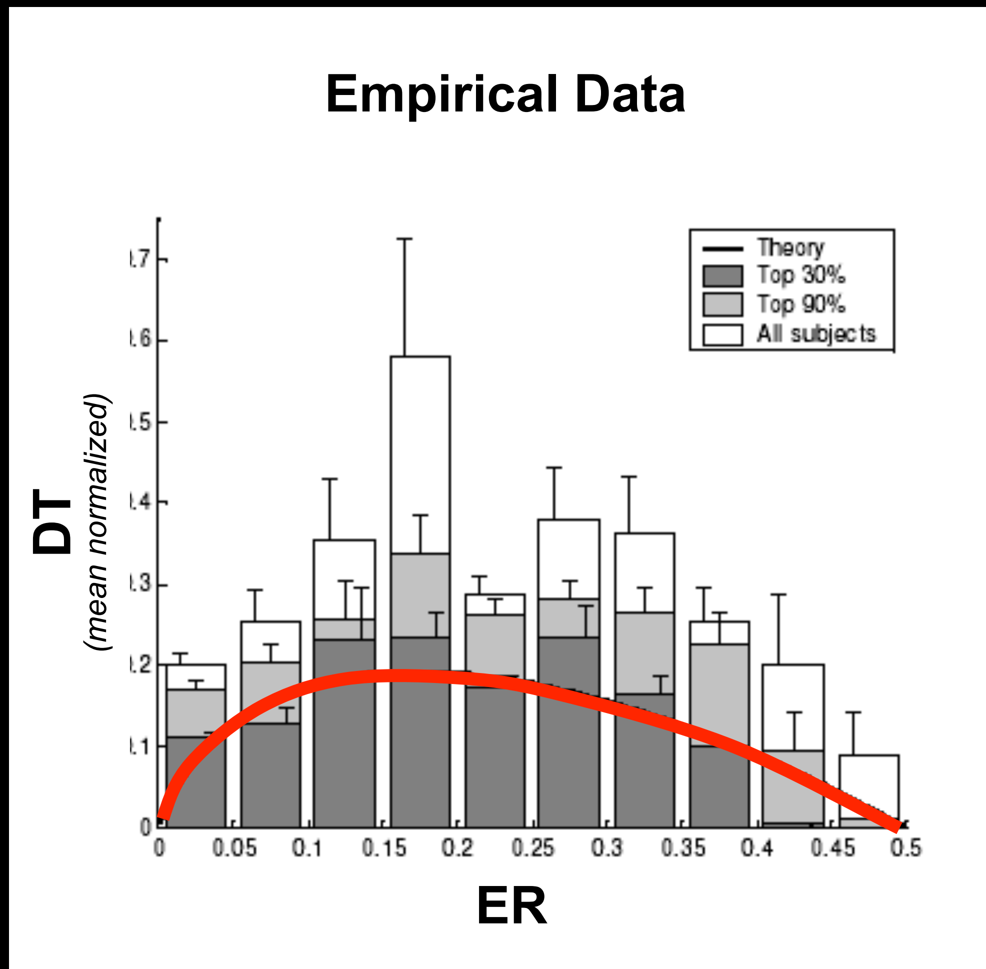

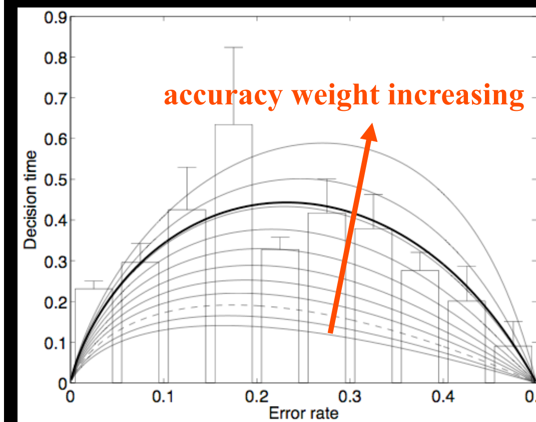

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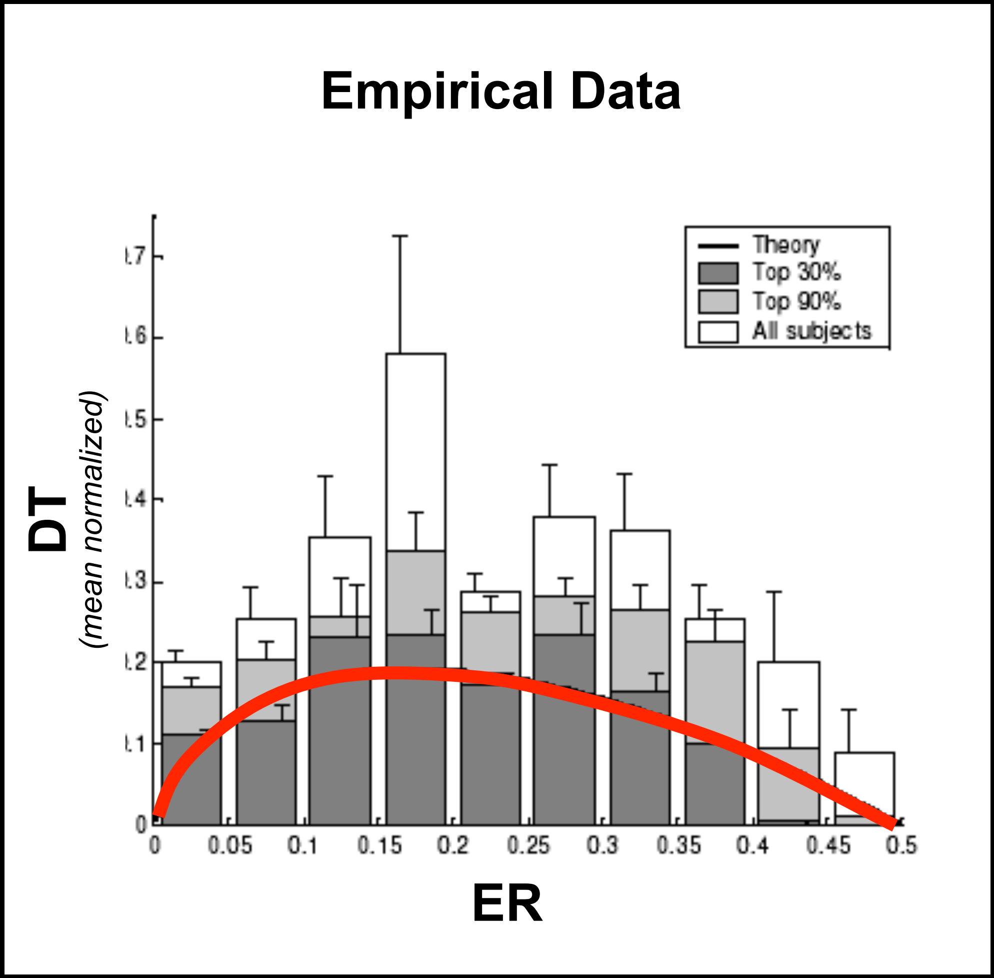

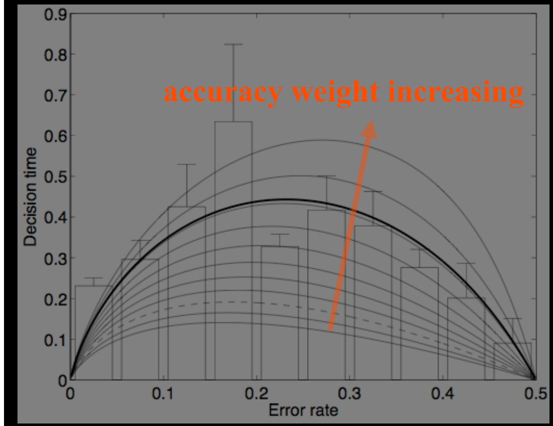

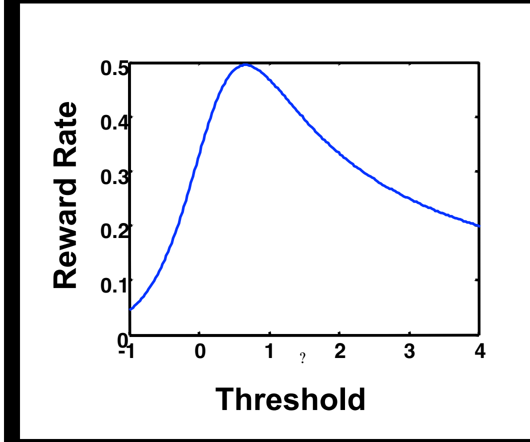

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  - **explain human RT distributions and accuracy in simple decision tasks**

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- **Drift Diffusion Model (DDM) can be used to:**
  - **explain dynamics of neural firing in simple decision tasks**

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- **Drift Diffusion Model (DDM) can be used to:**
  - **formally analyze neural network models of simple decision tasks**

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- **Drift Diffusion Model (DDM) can be used to:**
  - **describe parameters of optimal performance (maximizing reward rate)**

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- **Drift Diffusion Model (DDM) can be used to:**
  - **predict influence of task parameters on speed-accuracy tradeoff that approximate those observed empirically**

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- Drift Diffusion Model (DDM) can be used to:
  - define, in formal terms, the mechanisms underlying decision making in simple two alternative forced choice tasks

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- Drift Diffusion Model (DDM) can be used to:
  - define, in formal terms, variables that are subject to regulation by control mechanisms to optimize outcomes

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- **Drift Diffusion Model (DDM) can be used to:**

  - **be approximated by neurally-plausible mechanism:**
    leaky competitive accumulator (LCA)

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- **Drift Diffusion Model (DDM) can be used to:**

  - **makes contact with abstract, mathematical modeling of psychological function:** *(e.g. normative models based on Bayesian inference)*