Learning

- Unsupervised Learning
  - Hebbian Learning Rule
  - Pattern associator
  - Self-organized maps
  - Topographic structure
  - Pattern detectors
- Supervised Learning
  - Scalar (Reinforcement) Learning
    - ☞ Classical and Instrumental Conditioning
    - Sequential learning and Prediction
  - Vector-Based Learning
    - Generalized Delta Rule
    - Backpropagation
    - Deep Learning

Reinforcement Learning

- Conditioning
  - Simple Prediction
    Rescorla-Wagner Rule
  - Stimulus-Action Associations
    Actor-critic model, Q Learning
- Sequence Prediction
  - Method of Temporal Differences (TD)
  - Model-Free vs. Model-Based RL
- Challenges
  - Curse of dimensionality
    - *Hierarchical RL: policies and options*
    - *State space abstraction*
  - Explore-exploit
    - ***Meta-control***

Conditioning

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  - But the learning is still *associational*

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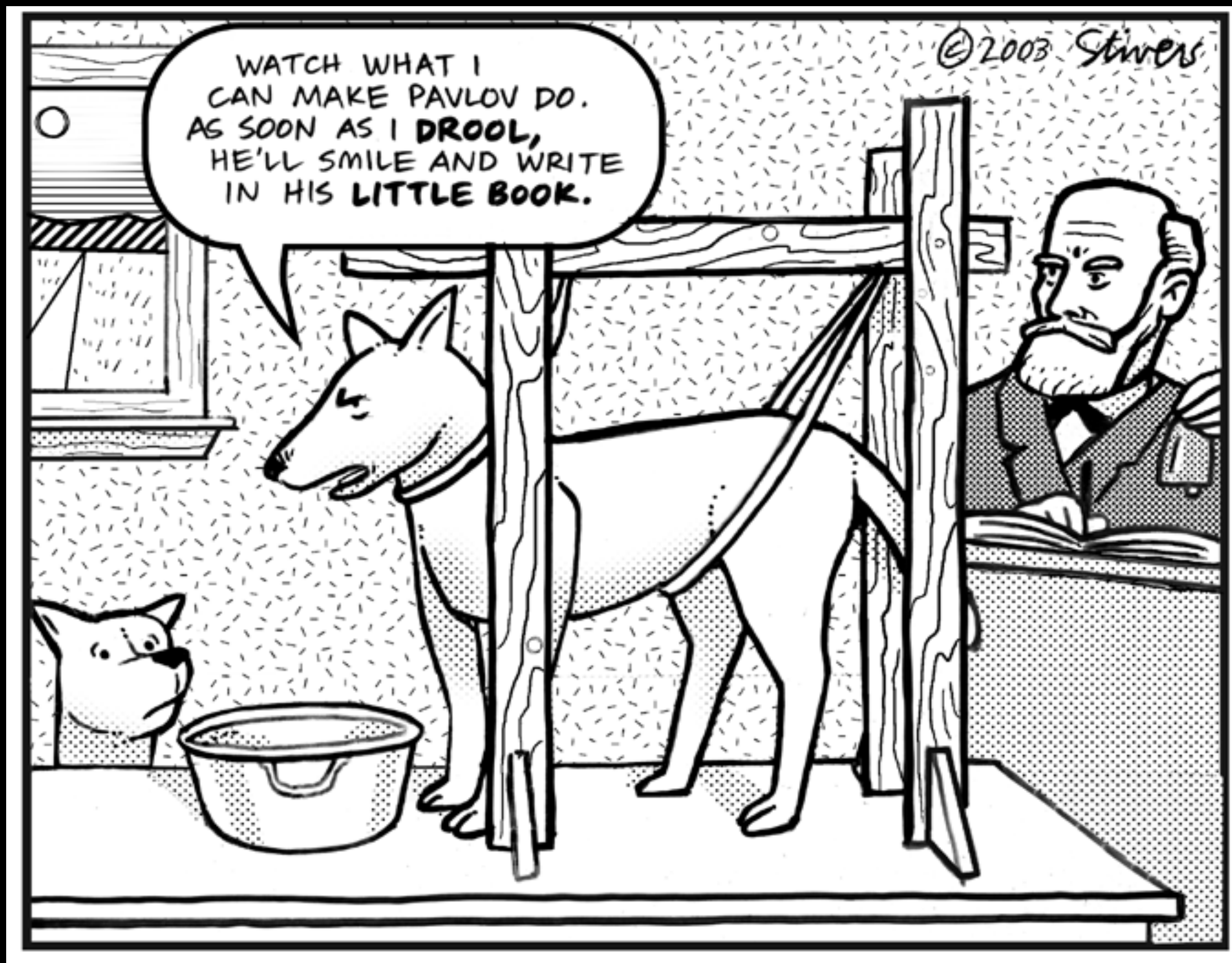
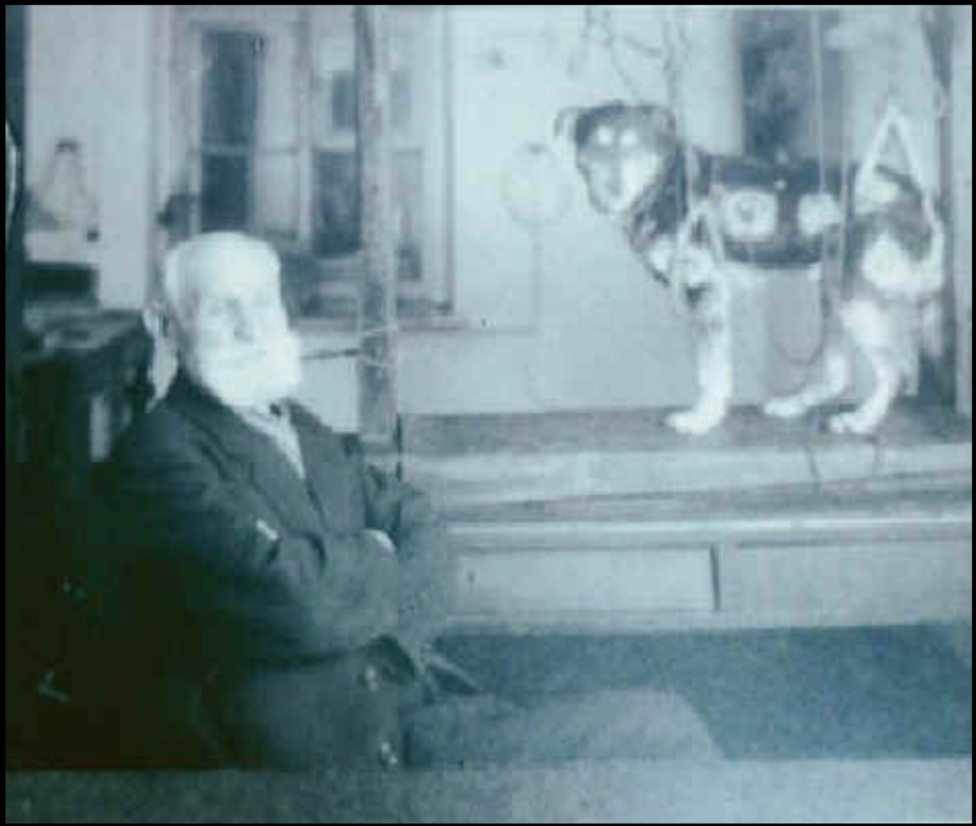
  - But the learning is still *associational*
- Based on similar principles of associative learning, but with a twist…

Conditioning

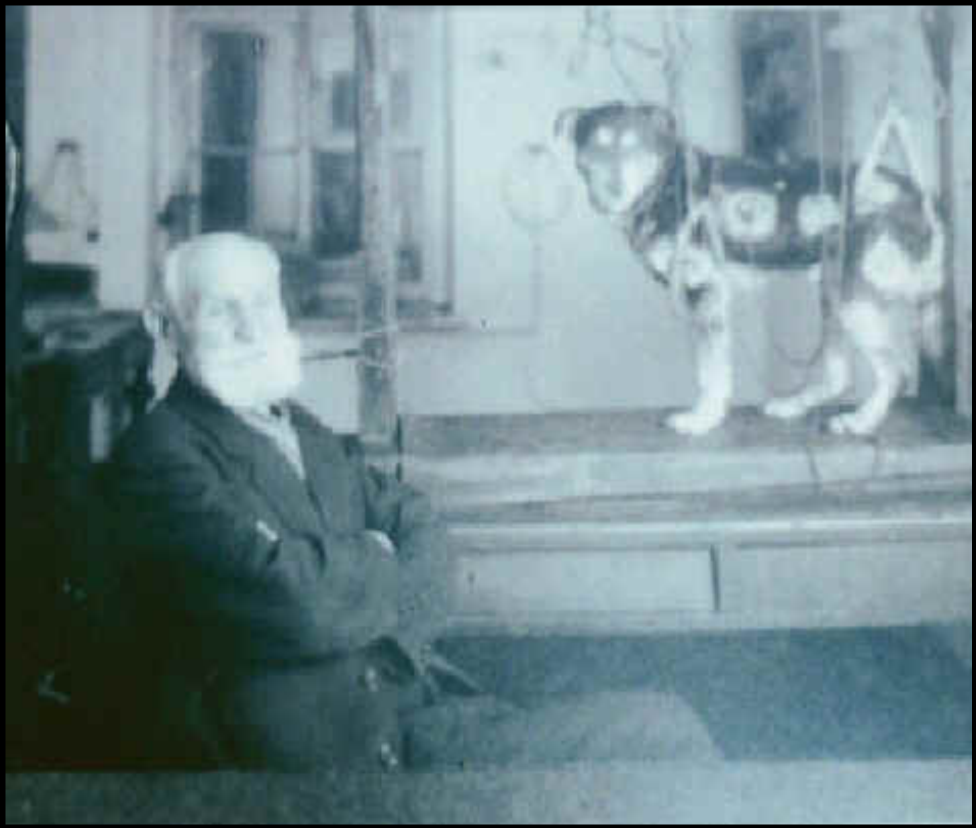


Classical (Pavlovian) Conditioning



pair stimulus (CS)

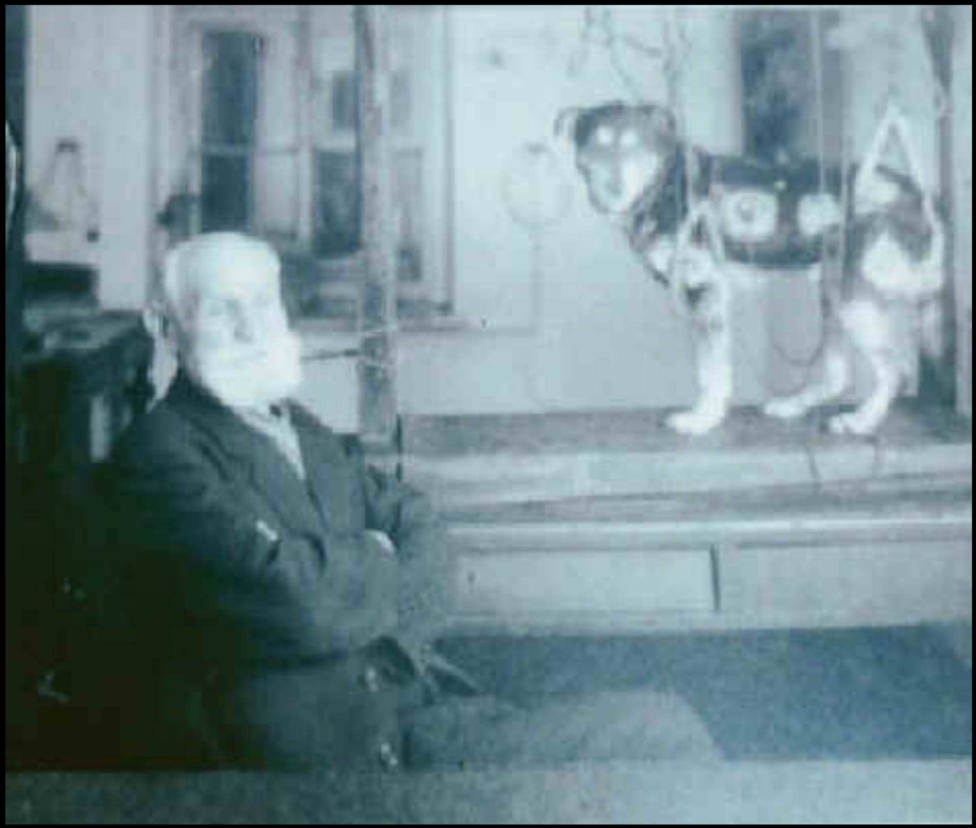
Classical (Pavlovian) Conditioning



pair stimulus (CS)

...with significant event (US)

Classical (Pavlovian) Conditioning



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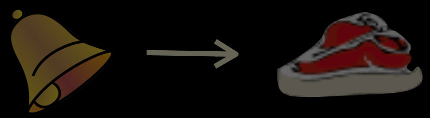
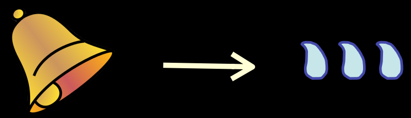


measure anticipatory behavior (CR)



Classical (Pavlovian) Conditioning

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Conditioned Stimulus (CS)

Classical (Pavlovian) Conditioning

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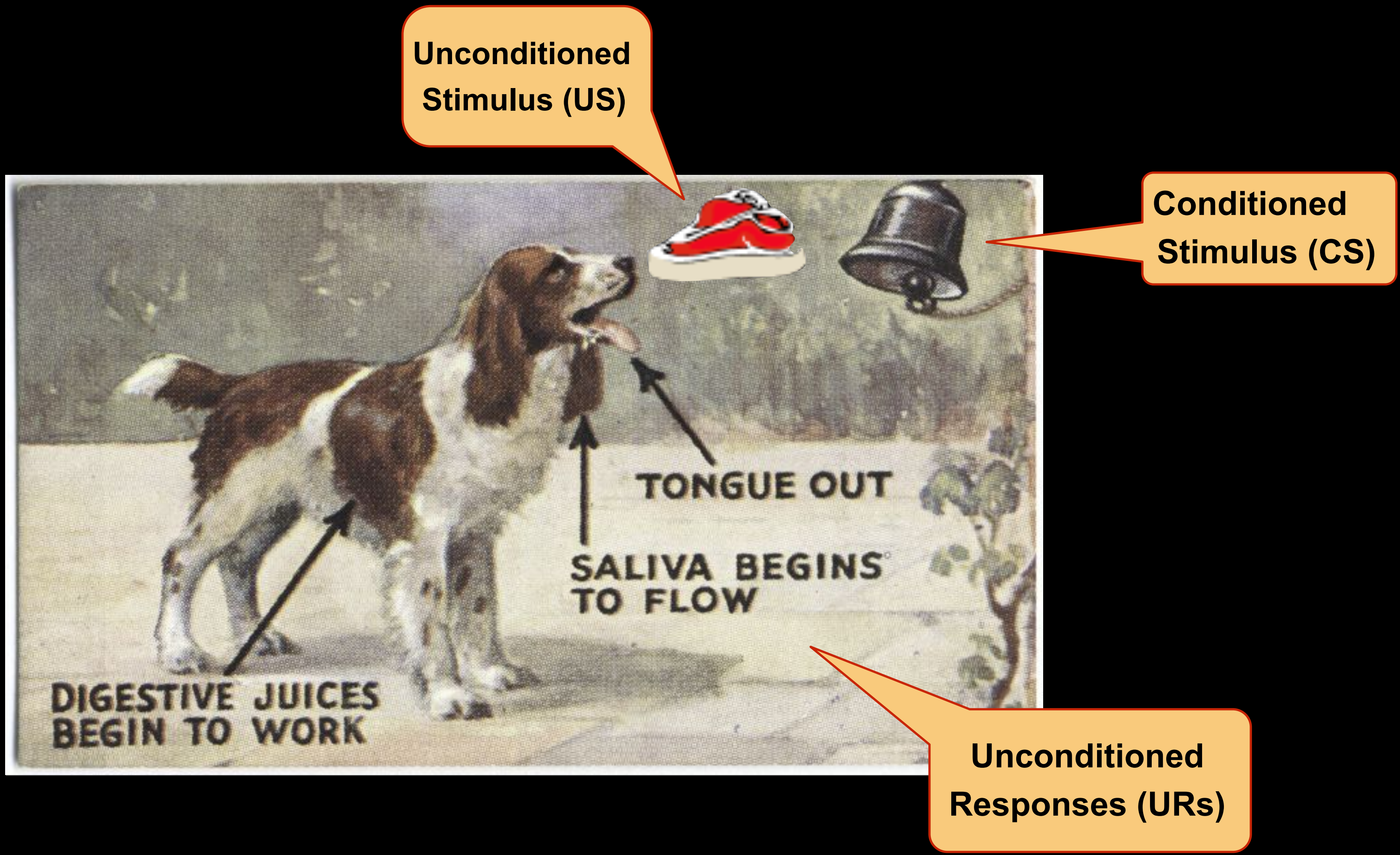
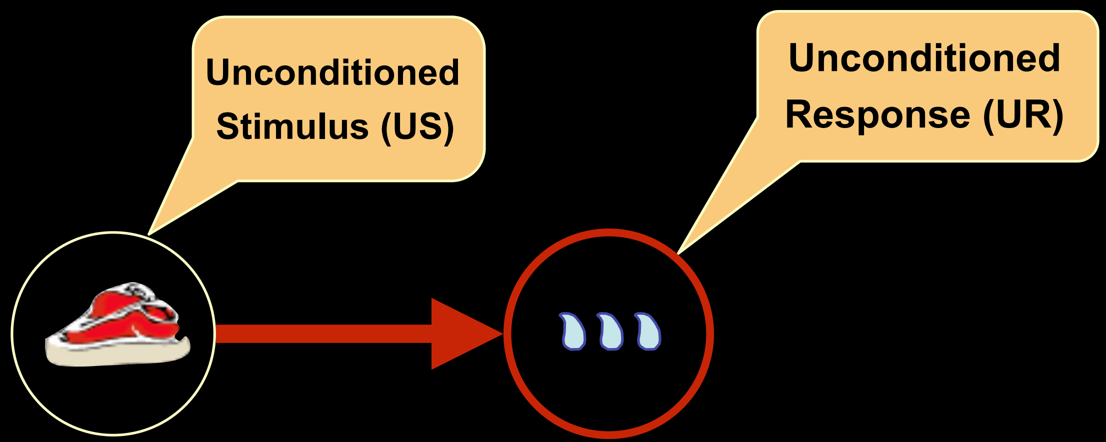


Conditioning



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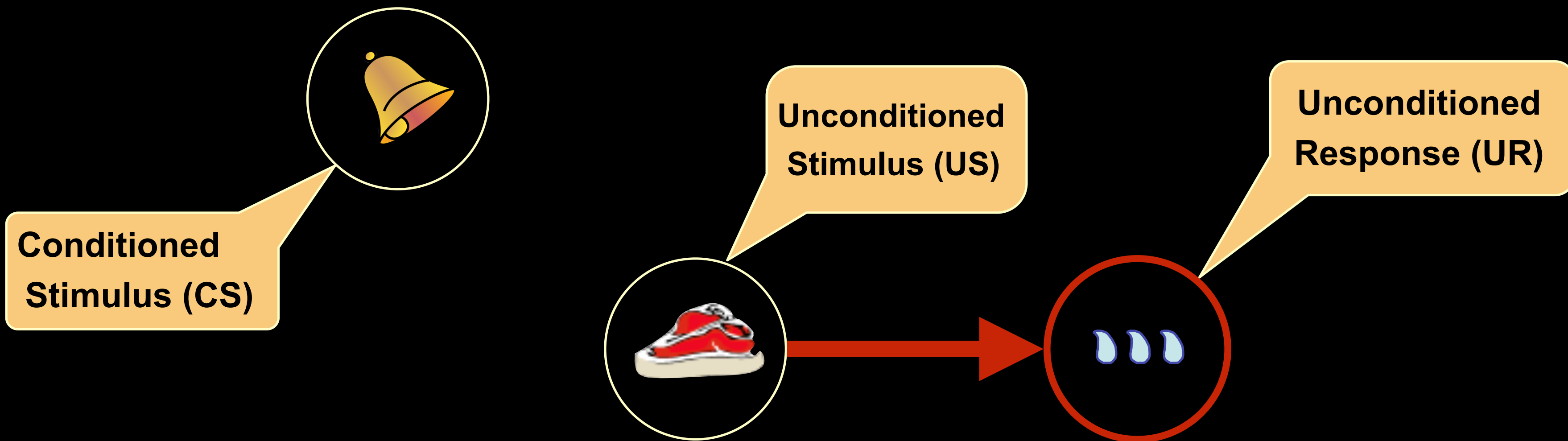

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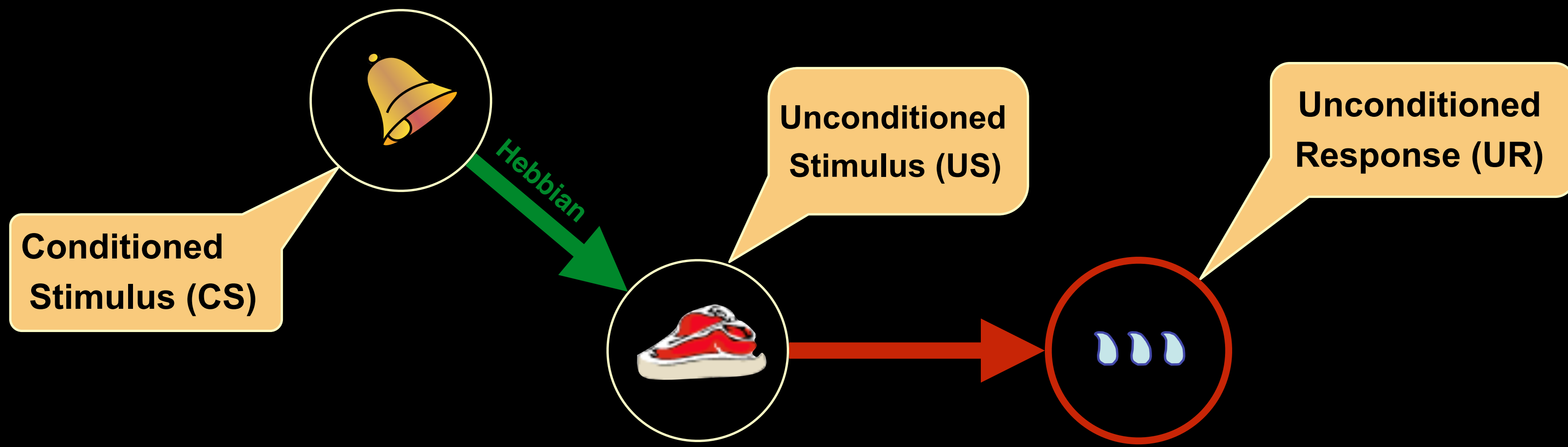

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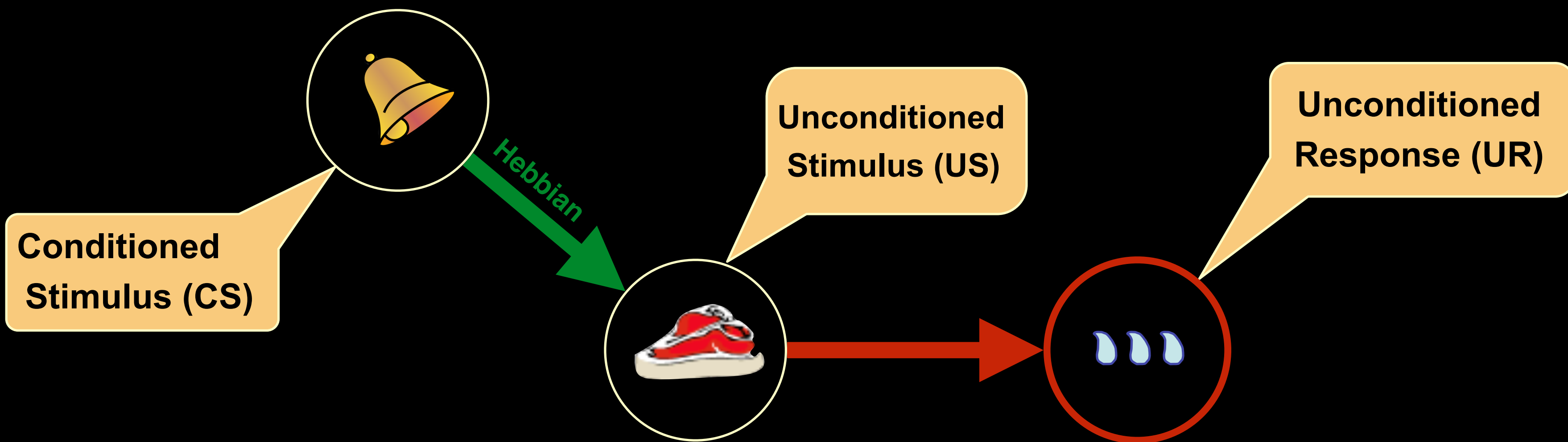
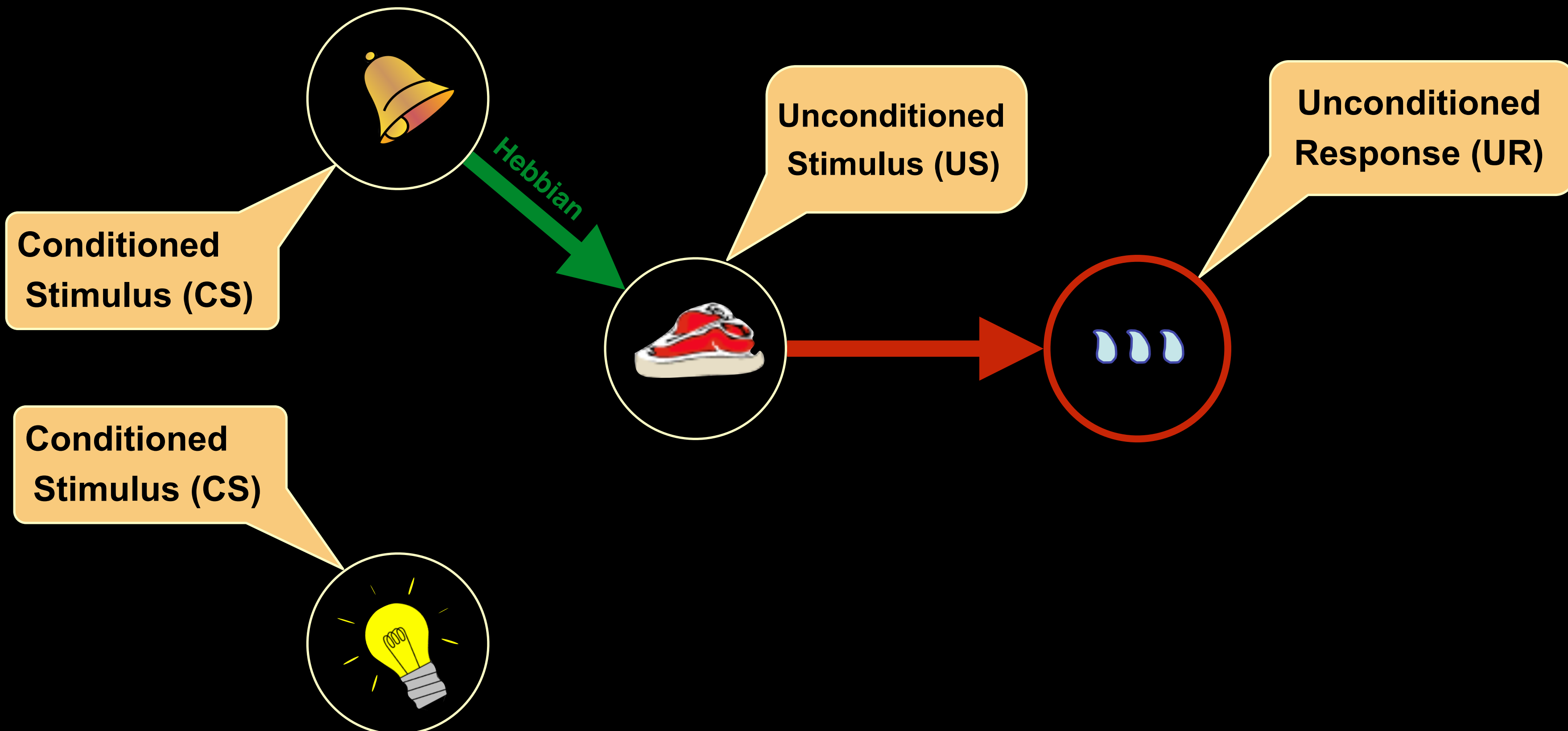


Pure Hebbian learning is purely about *contiguity*

Conditioning



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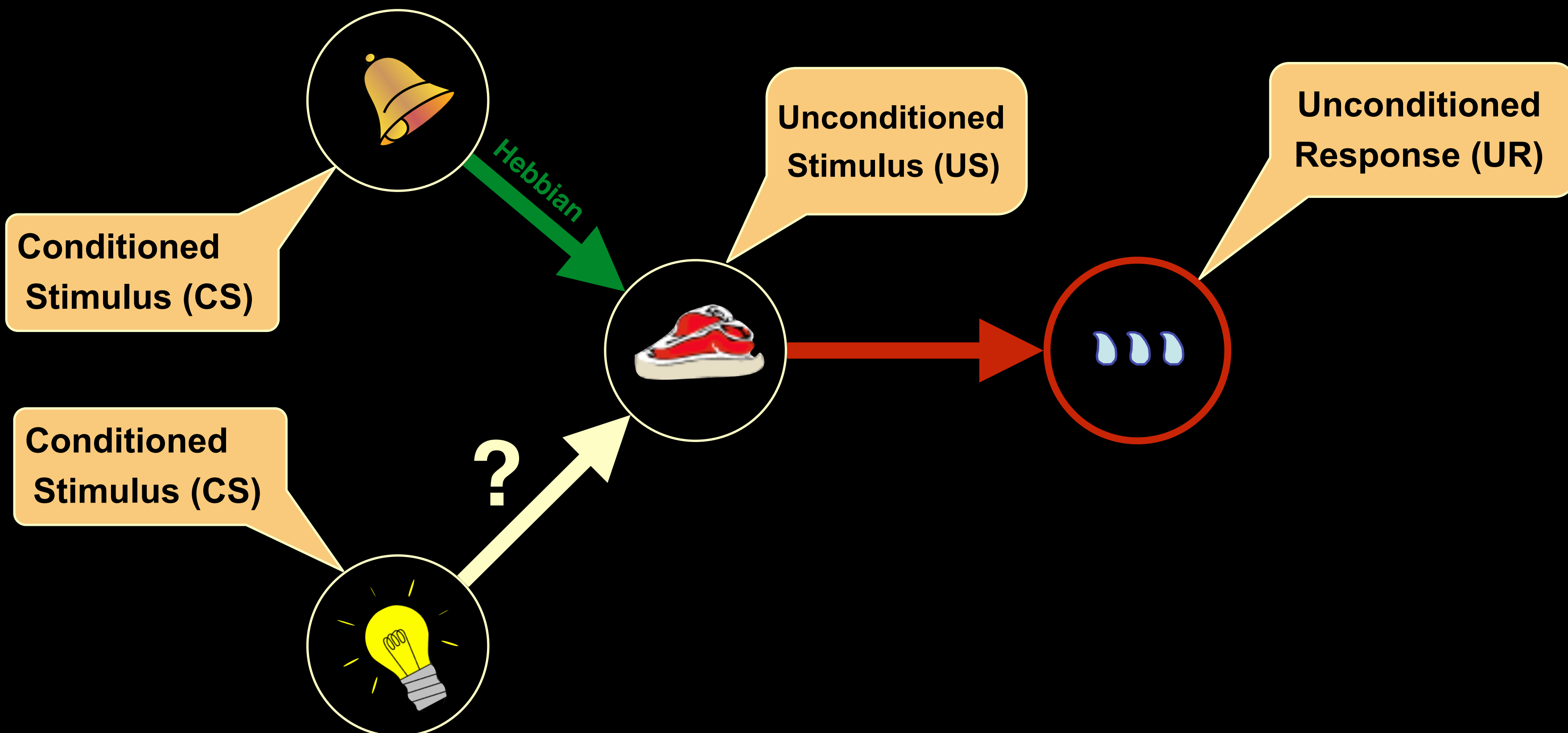

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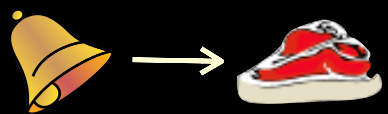
Blocking

Kamin (1968)

Phase I



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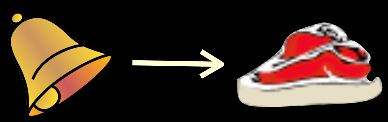
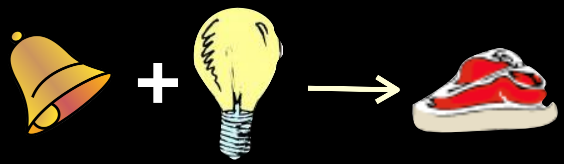


Phase II



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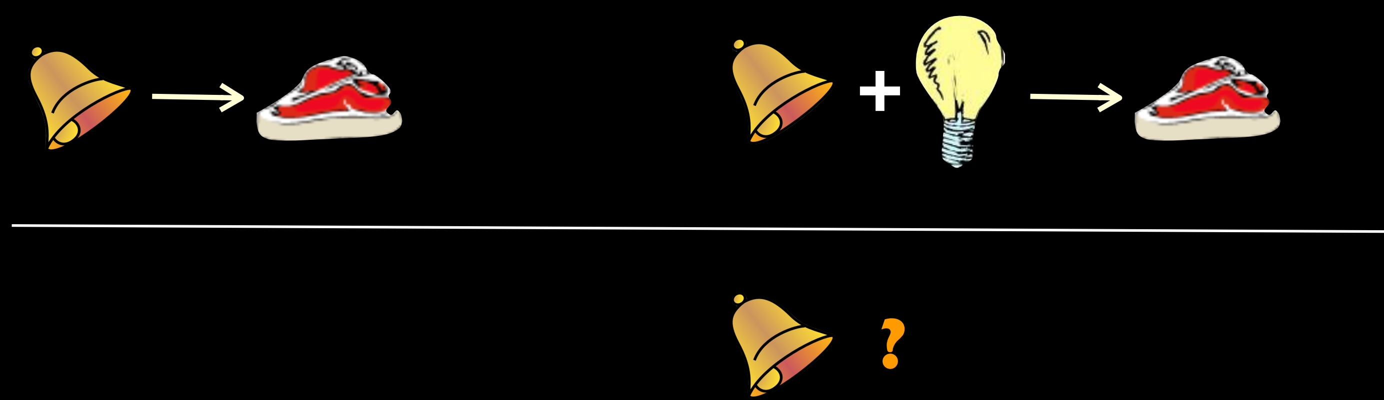
Phase I **Phase II**



Blocking

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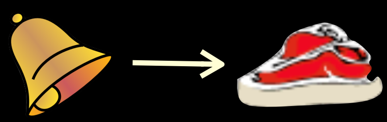
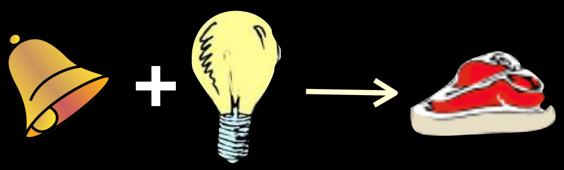
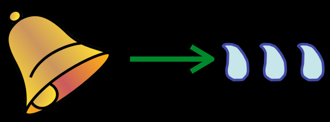
Phase II







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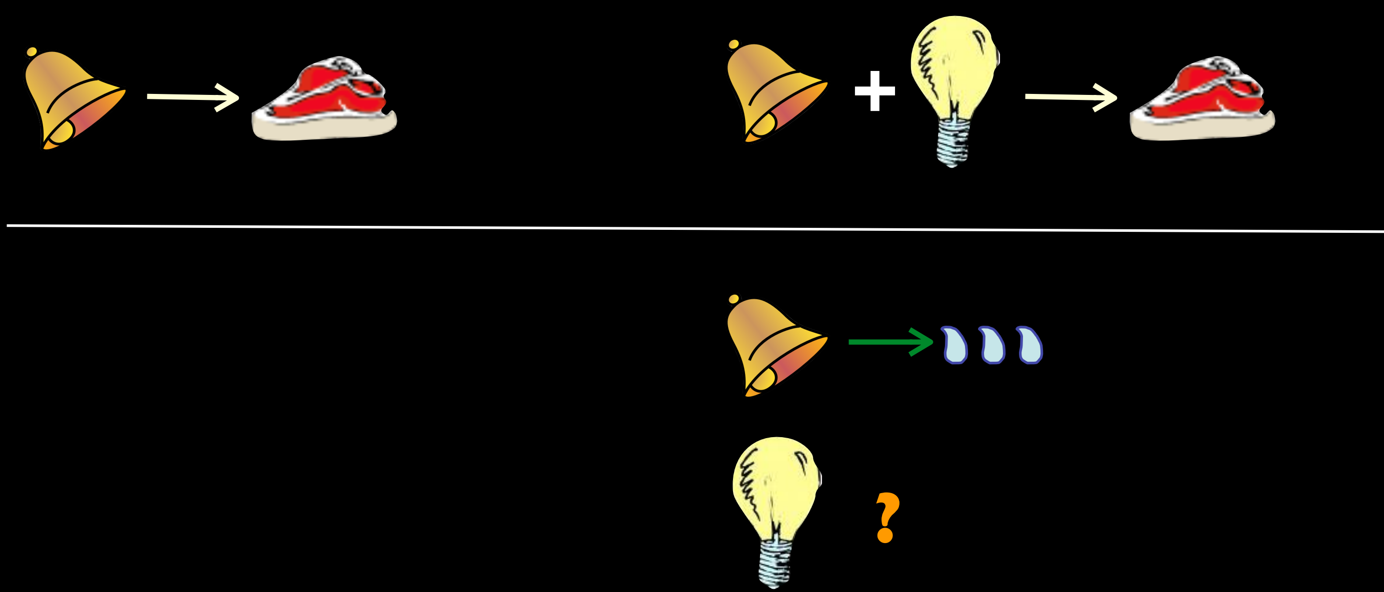

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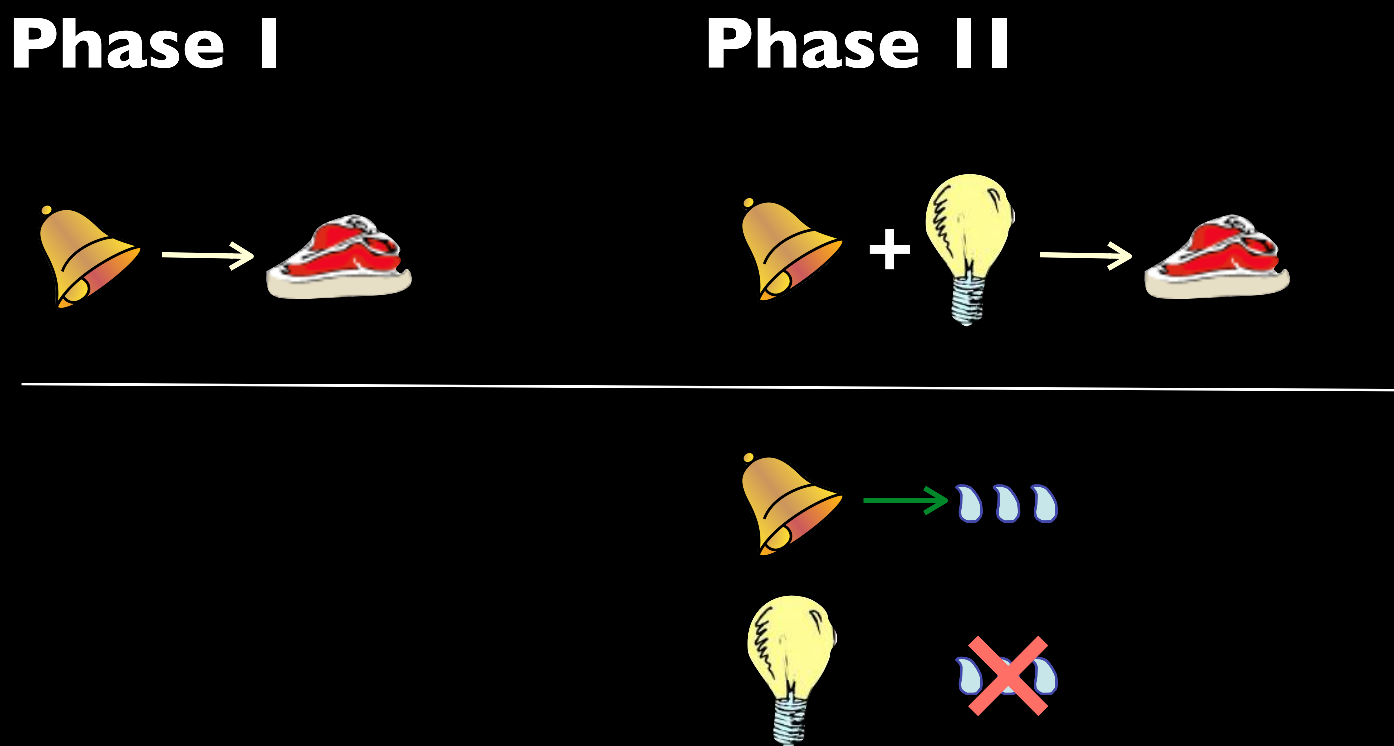

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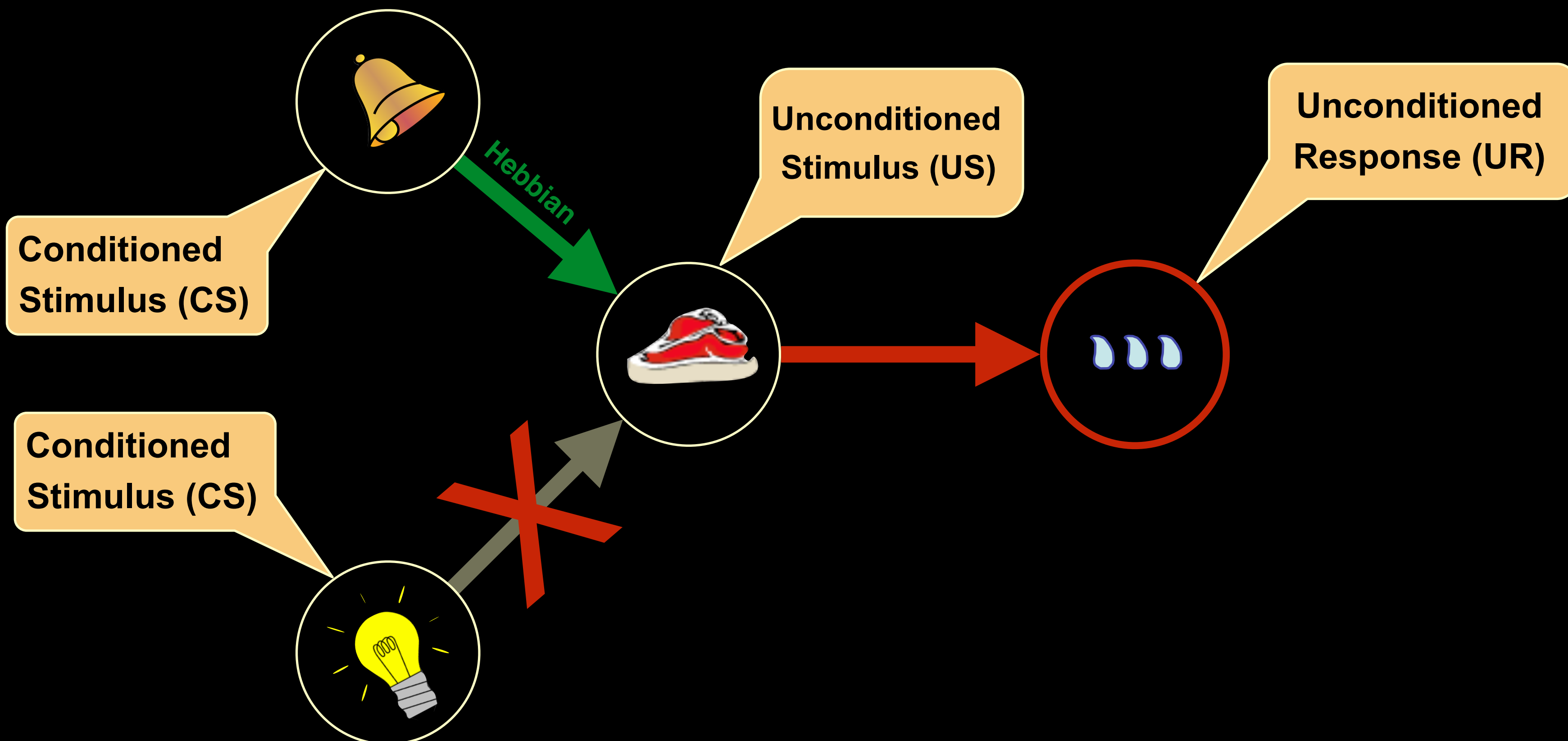


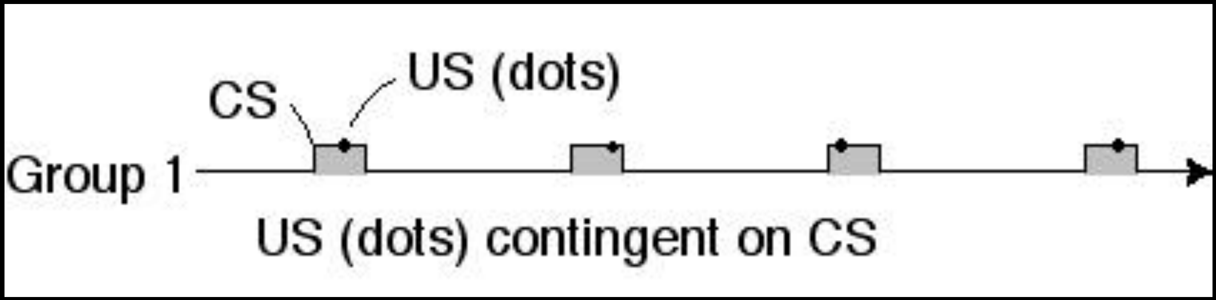
Conditioning must be about more than *just contiguity*…

Contingency

Contingency

Rescorla's experiment:

Standard



Contingency

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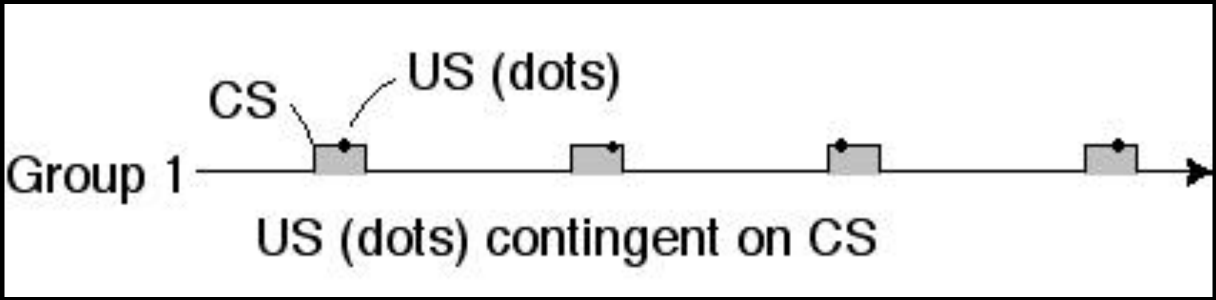

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Contingency

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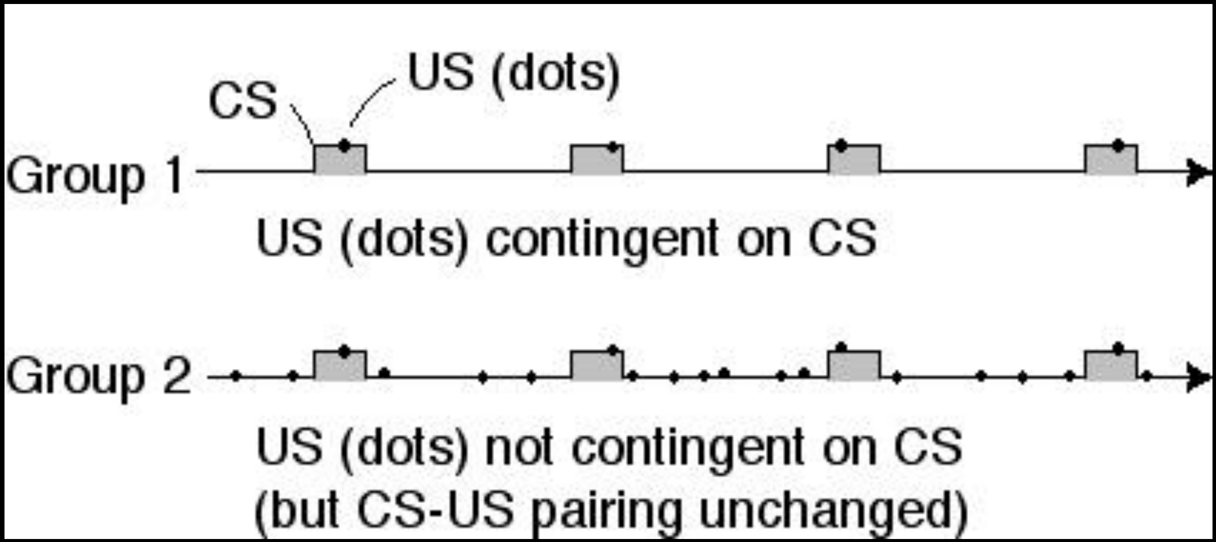

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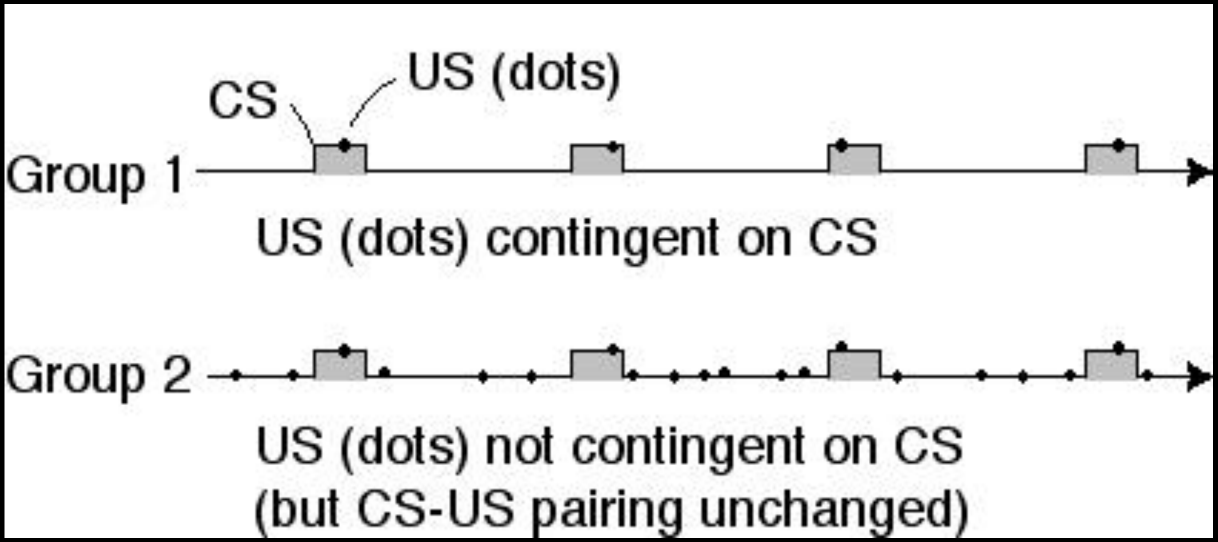

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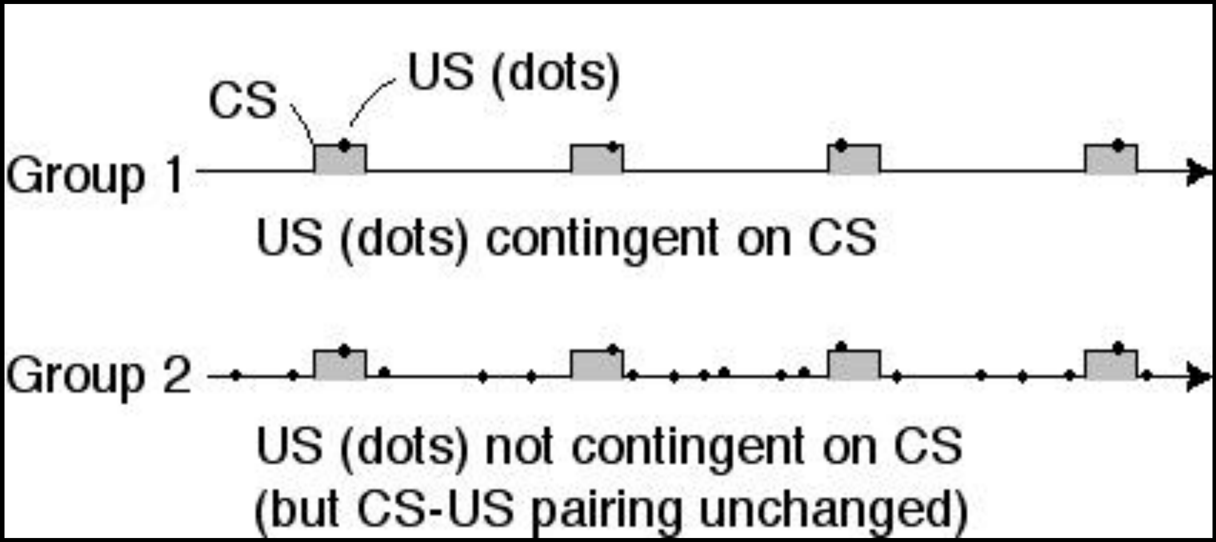
X

Contingency

Conditioning:

Standard

Random control



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X

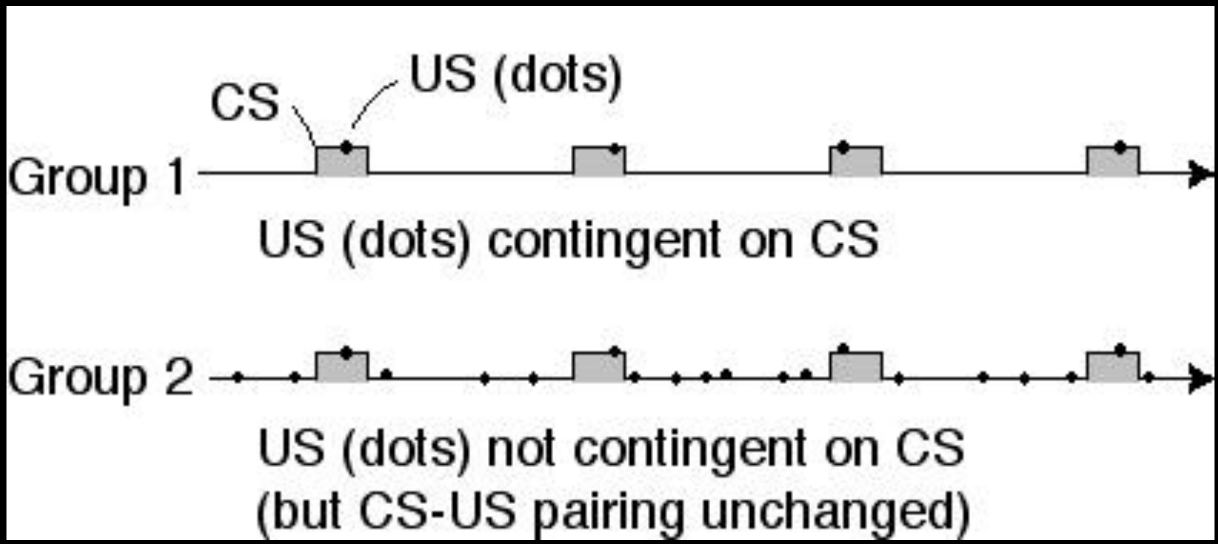
Temporal contiguity is not enough, need *contingency*

Contingency

Conditioning:

Standard

Random control



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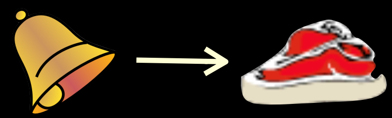
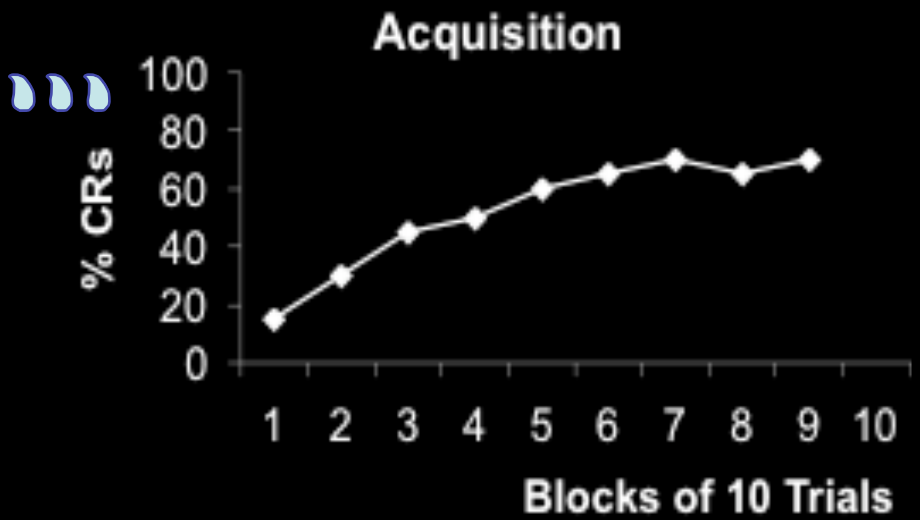
X

Contingency ⇒ *Prediction*

Contingency

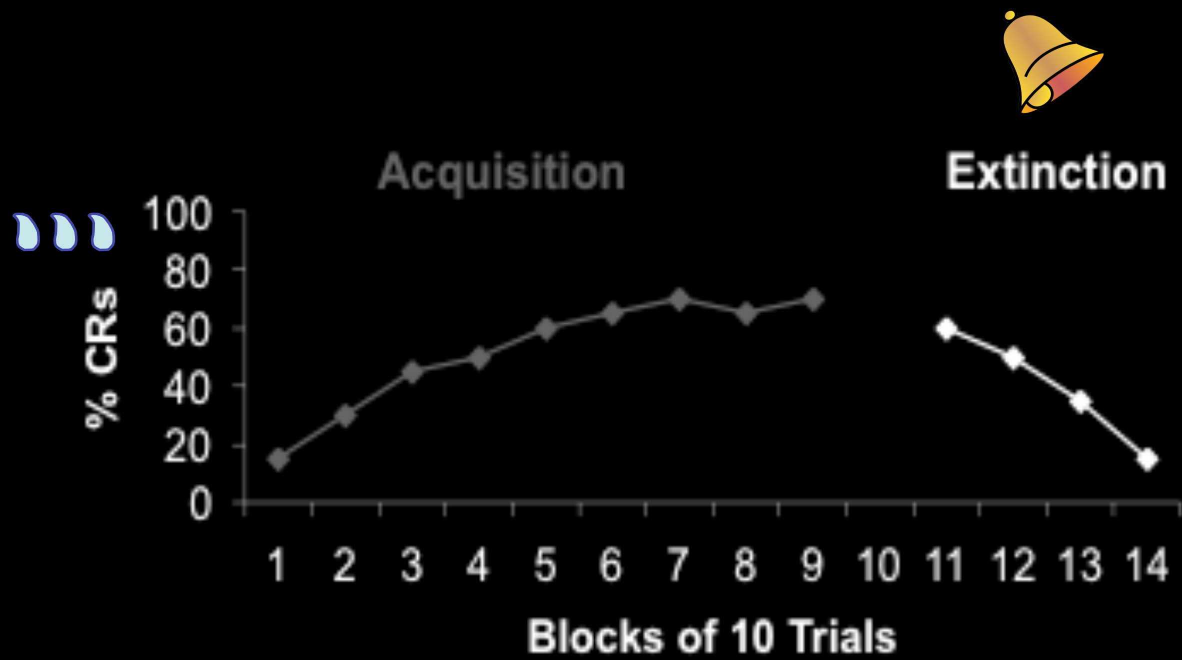
Prediction





Contingency

Failure of *Prediction*



Prediction Learning

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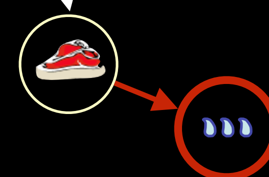

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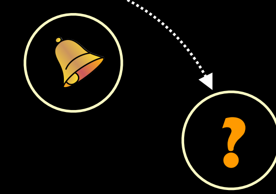

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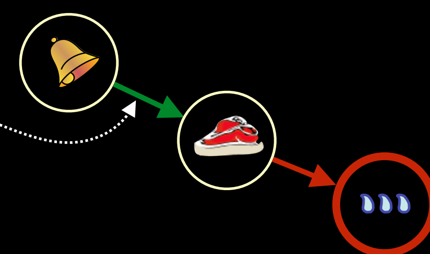

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w_t is the strength of the association between the CS and US

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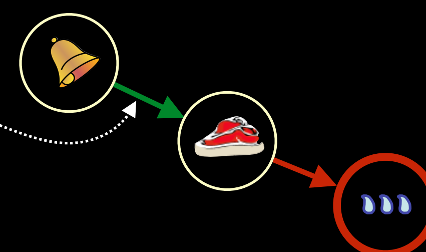

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v_t is the predicted value of the CS
Δv is the reward prediction error
w_t is the strength of the association between the CS and US
α is the learning rate

Reward Prediction

Prediction error

$$\Delta v = r_t - v_t$$

Reward Prediction

Prediction error

$$\Delta v = r_t - v_t$$

Trial 1

Reward Prediction

Prediction error

$$\Delta v = r_t - v_t$$

Trial 1

Reward Prediction

Prediction error

$$\Delta v = r_t \;-\; v_t$$

Trial 1

$$\begin{bmatrix} 0 & \quad & \quad \\ & & \end{bmatrix}$$

Reward Prediction

Prediction error

$$\Delta v = r_t - v_t$$

Trial 1

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Reward Prediction

Prediction error

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Trial 1

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Reward Prediction

Prediction error

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Trial 1

$$+ \left[\; 0 \; \right]$$

Reward Prediction

Prediction error

$$\Delta v = r_t - v_t$$

Trial 1

+ +

$$\left[\ 0\ \right]$$

Reward Prediction

Prediction error | *Weight adjustment*

$$\Delta v = r_t - v_t \qquad w_t + \alpha\, \Delta v$$

Trial 1

$$+ \quad + \quad \begin{bmatrix} 0 & 0 \\ & 0 \end{bmatrix} \quad +$$

Reward Prediction

Prediction error *Weight adjustment*

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Trial 1

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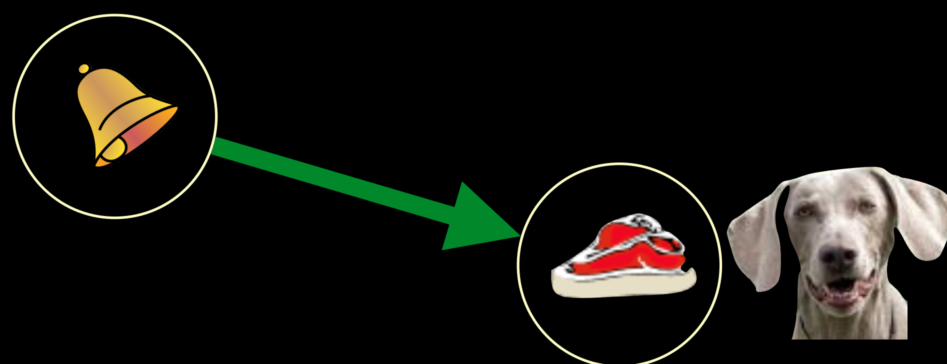

Reward Prediction

Prediction error | *Weight adjustment*

$\Delta v = r_t - v_t$

$w_{t+1} + \alpha\, \Delta v$

 $\left[\;\; + \;\;\right]$

 $\left[\;\; 0 \;\;\right]$

Reward Prediction

Prediction error | *Weight adjustment*

$\Delta v = r_t - v_t \qquad w_{t+2} = w_{t+1} + \alpha \, \Delta v$

Trial 2

$$\begin{bmatrix} + \\ 0 \end{bmatrix}$$

Reward Prediction

Prediction error *Weight adjustment*

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Reward Prediction

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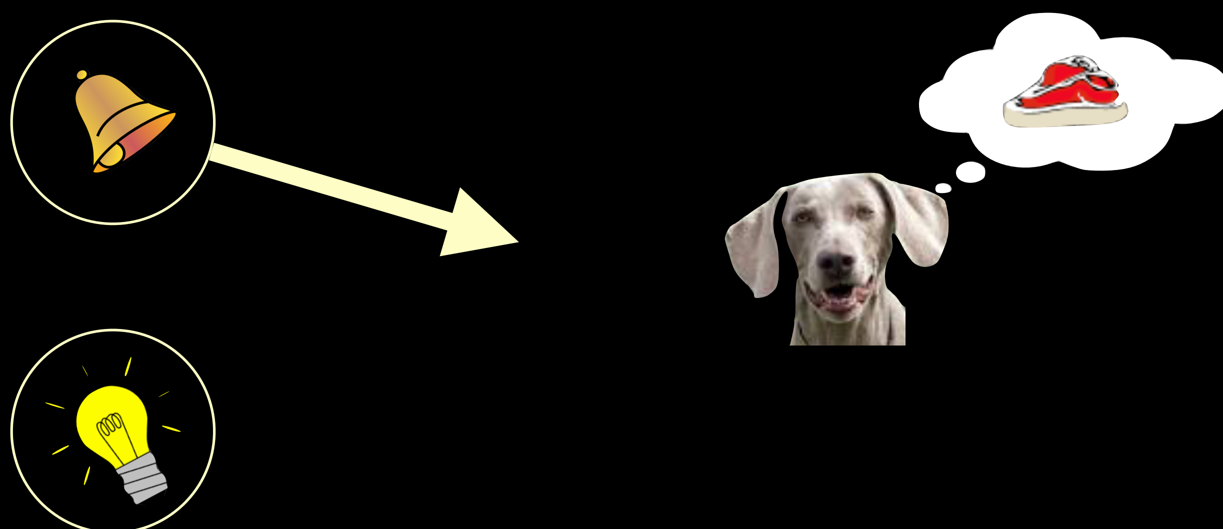

Reward Prediction

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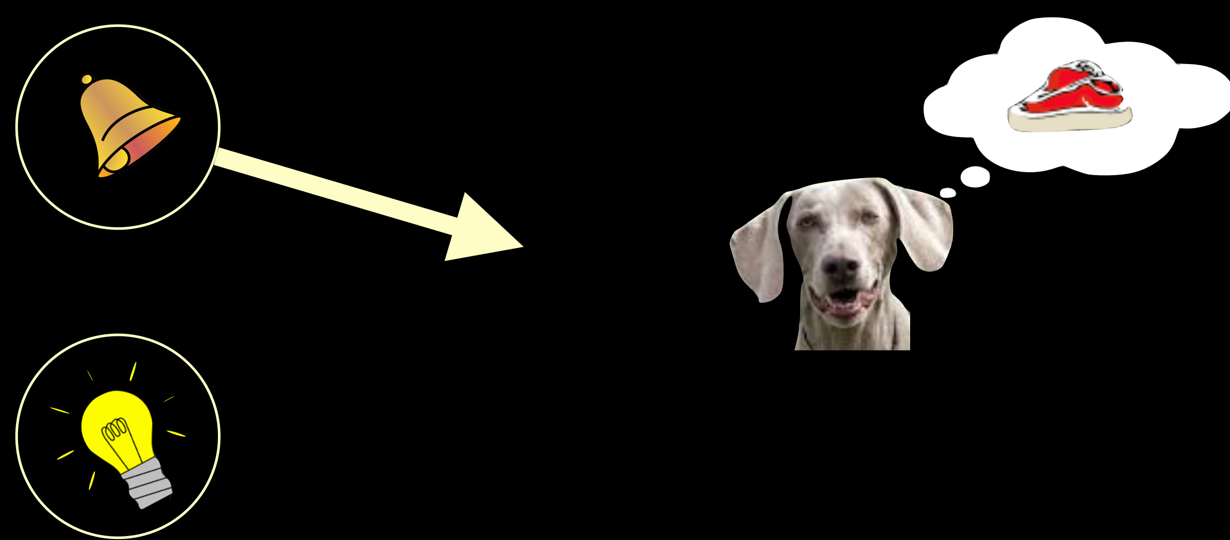

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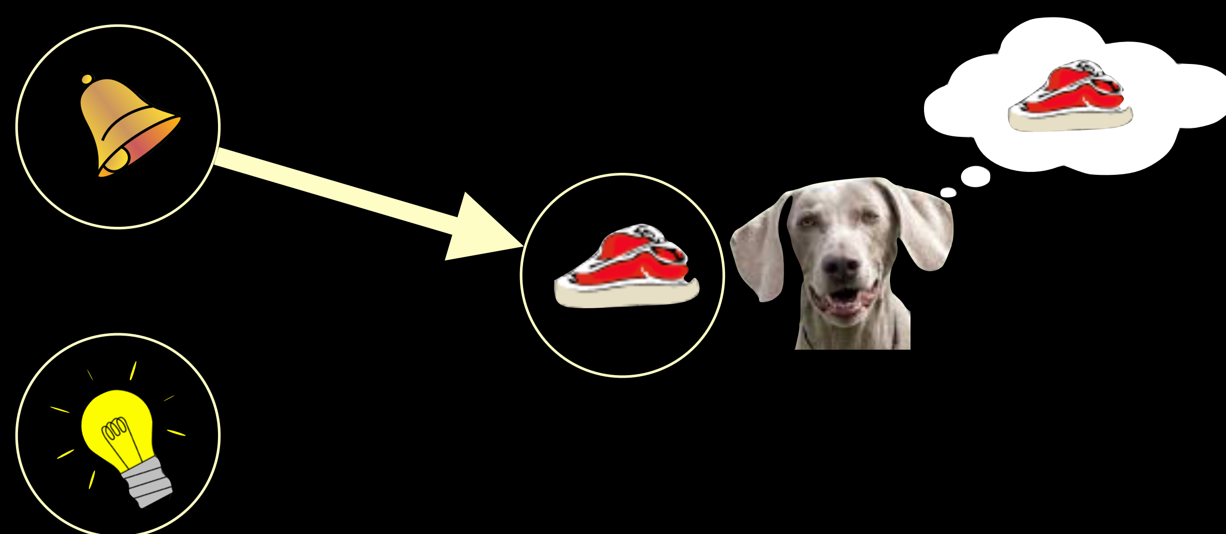

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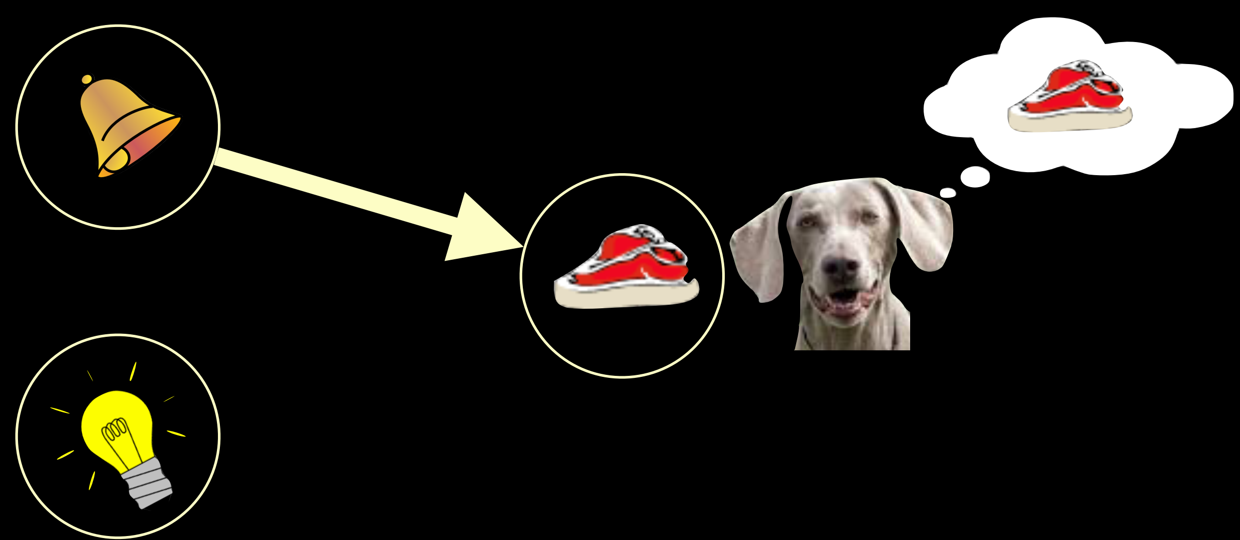

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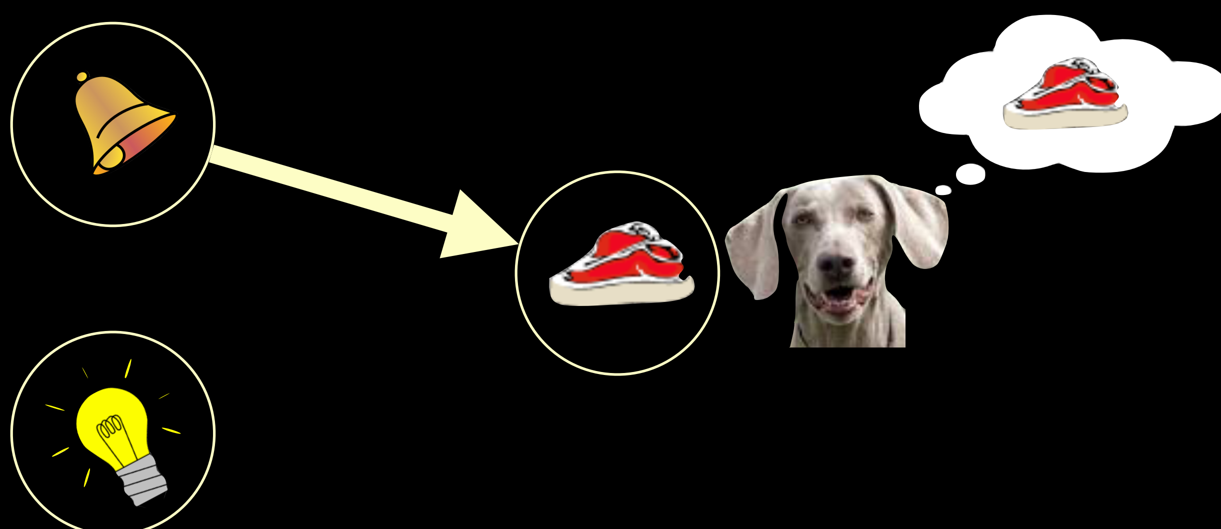

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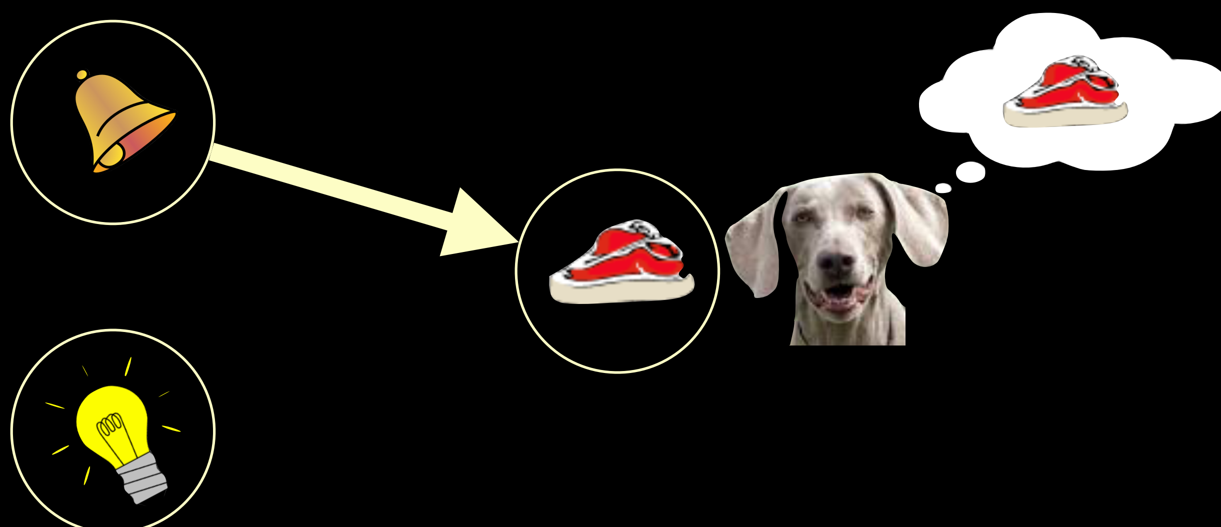

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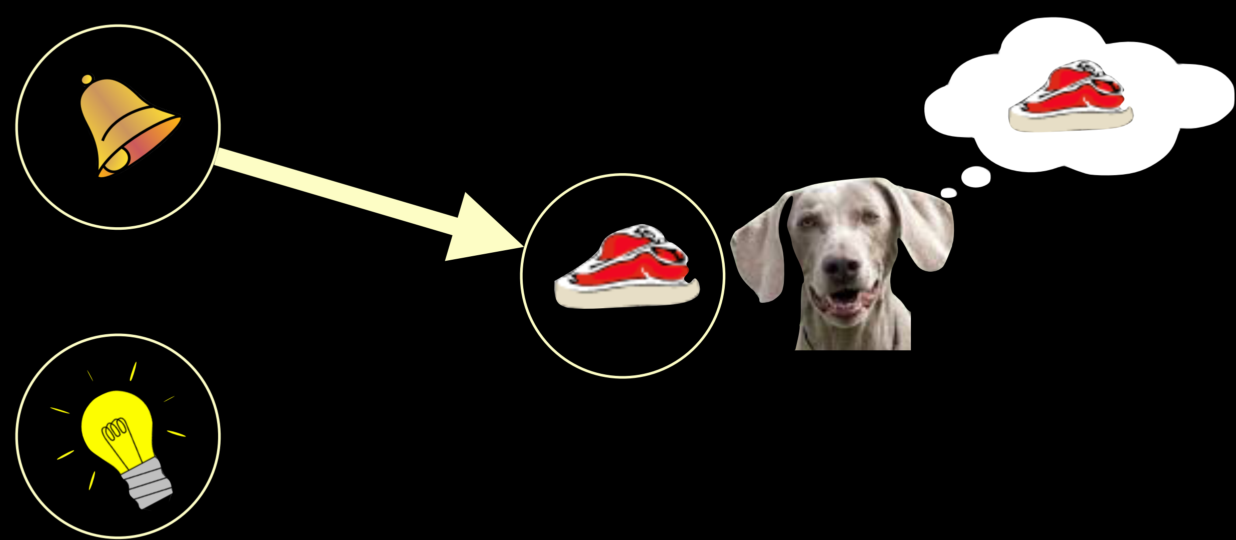
Trial 2

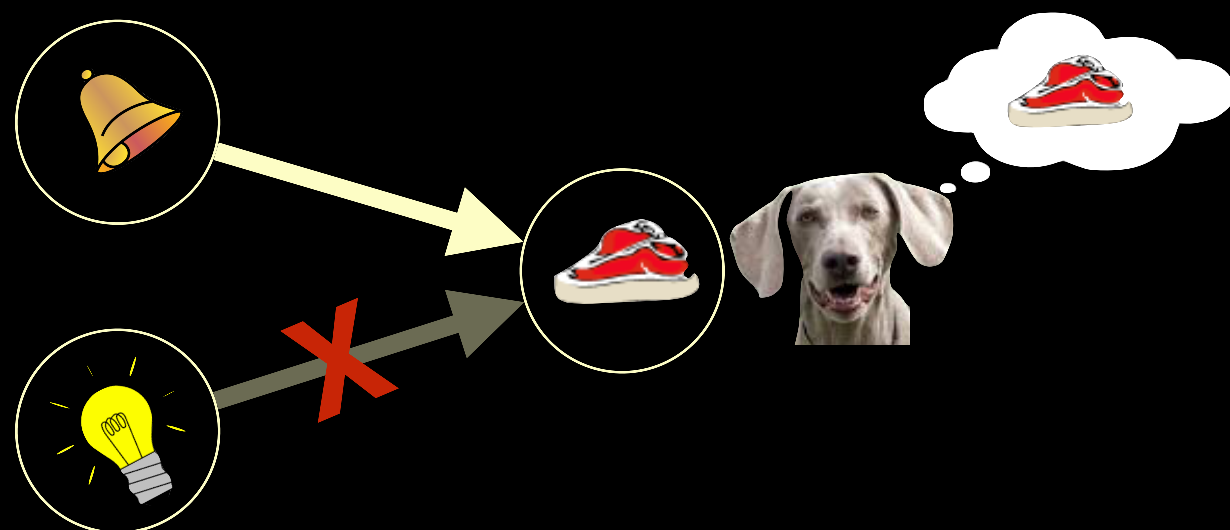
$$0 \quad + \quad \begin{bmatrix} + & + & + \\ 0 & 0 & 0 \end{bmatrix} \quad 0$$



Reward Prediction

Prediction error	*Weight adjustment*
$\Delta v = r_t - v_t$	$w_{t+2} = w_{t+1} + \alpha \, \Delta v$

Trial 2 $\quad 0 \; + \; \begin{bmatrix} + & + & + \\ 0 & 0 & 0 \end{bmatrix} \; 0$



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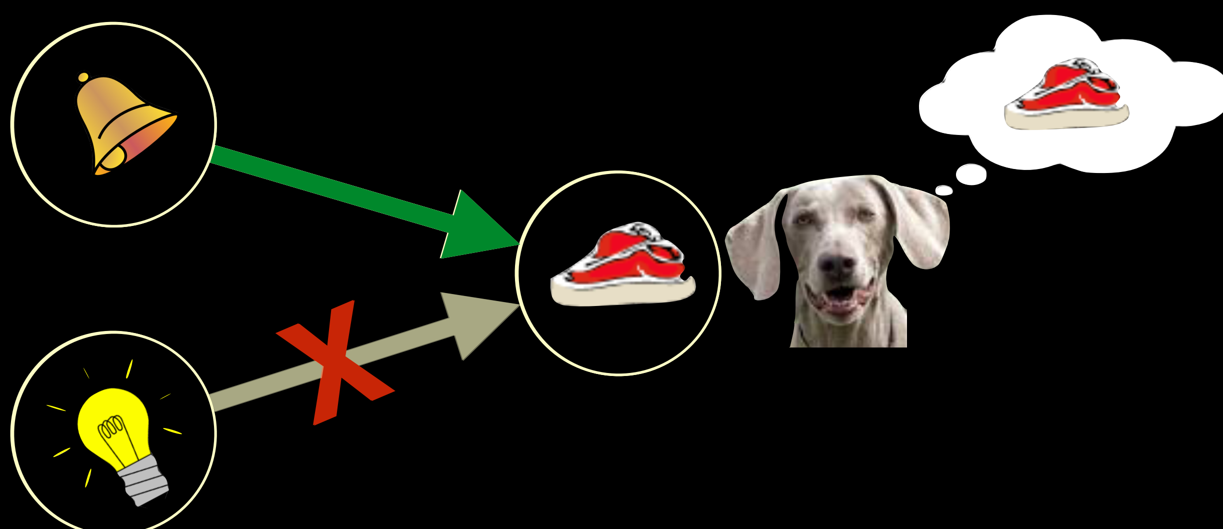
Trial 2

$$0 \quad + \quad \begin{bmatrix} + & + & + \\ 0 & 0 & 0 \end{bmatrix} \quad 0$$



Prediction Learning

- **Rescorla-Wagner Learning Rule**

Prediction error

$$\Delta w_S = r_{S(t)} - w_{S(t)}$$

Weight adjustment

$$w_{S(t+1)} = w_{S(t)} + \alpha \, \Delta w_S$$

Prediction Learning

- **Rescorla-Wagner Learning Rule**

Prediction error ***Weight adjustment***

$$\alpha \quad r_{S(t)} - w_{S(t)}$$

Prediction Learning

- **Rescorla-Wagner Learning Rule**

Prediction error ***Weight adjustment***

$$w_{S(t+1)} = w_{S(t)} + \alpha r_{S(t)} - \alpha w_{S(t)})$$

Prediction Learning

- **Rescorla-Wagner Learning Rule**

Prediction error ***Weight adjustment***

$$w_{S(t+1)} = w_{S(t)} - \alpha w_{S(t)} \;\; + \alpha r_{S(t)})$$

Prediction Learning

- **Rescorla-Wagner Learning Rule**

Prediction error ***Weight adjustment***

$$w_{s(t+1)} = (1-\alpha)\, w_{s(t)} + \alpha\, r_{s(t)}\,)$$

Prediction Learning

- **Rescorla-Wagner Learning Rule**

Prediction error

Weight adjustment

$$w_{s(t+1)} = (1-\alpha)\, w_{s(t)} + \alpha\, r_{s(t)}\,)$$

Learn to predict rewards by averaging:

Prediction Learning

- Rescorla-Wagner Learning Rule

Prediction error

Weight adjustment

$$w_{S(t+1)} = (1-\alpha)\, w_{S(t)} + \alpha\, r_{s(t)})$$

Learn to predict rewards
by averaging: learned predictions

Prediction Learning

- Rescorla-Wagner Learning Rule

Prediction error

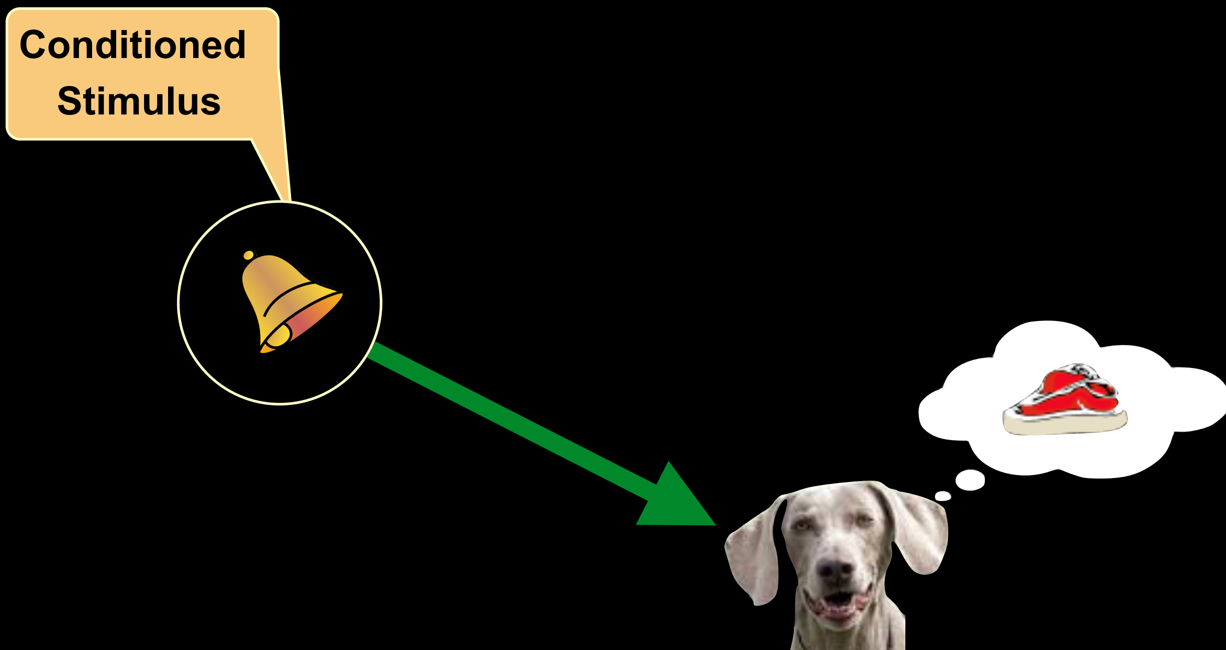
Weight adjustment

$$w_{S(t+1)} = (1-\alpha)\, w_{S(t)} + \alpha\, r_{S(t)}\,)$$

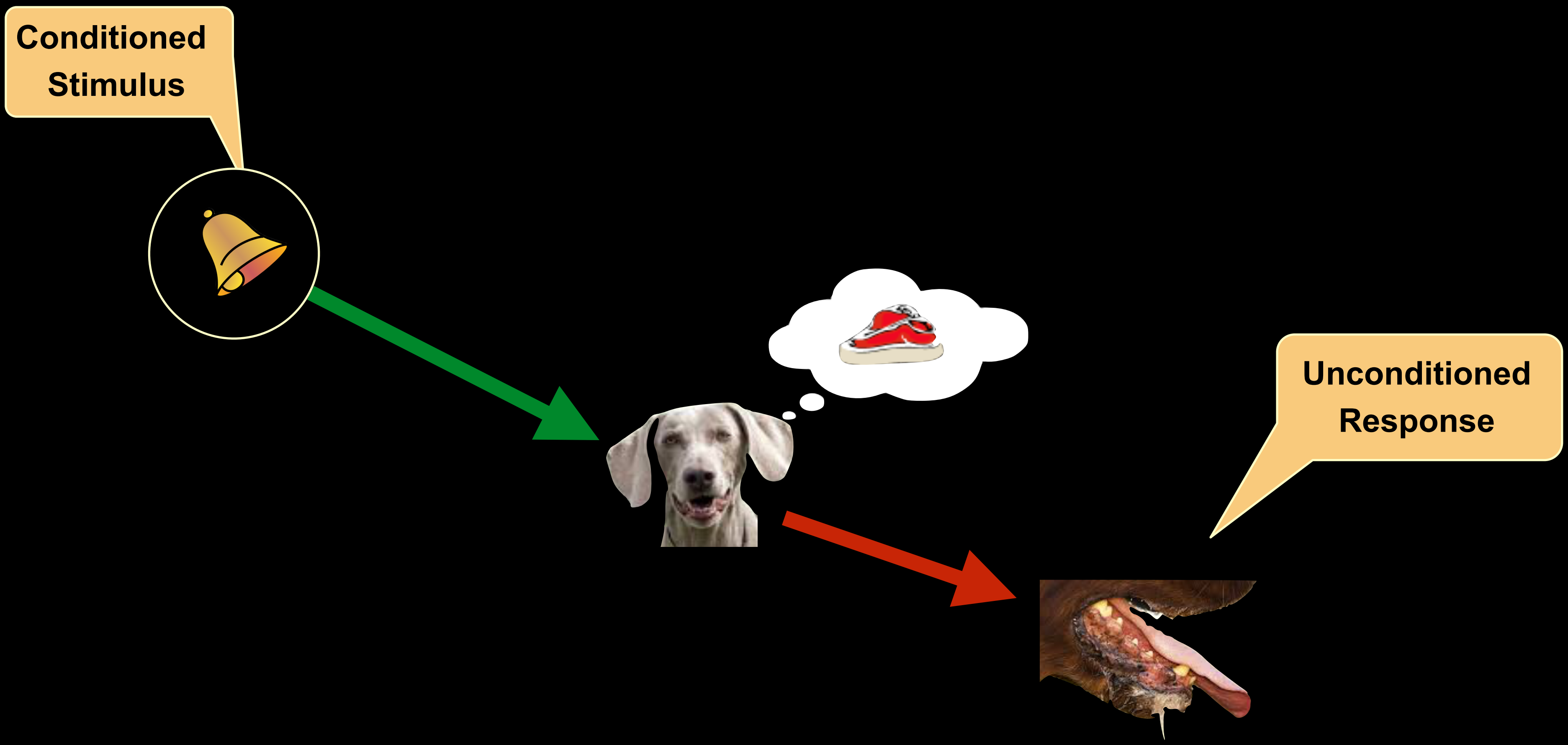
Learn to predict rewards
by averaging: learned predictions with present reward

Conditioning and Prediction

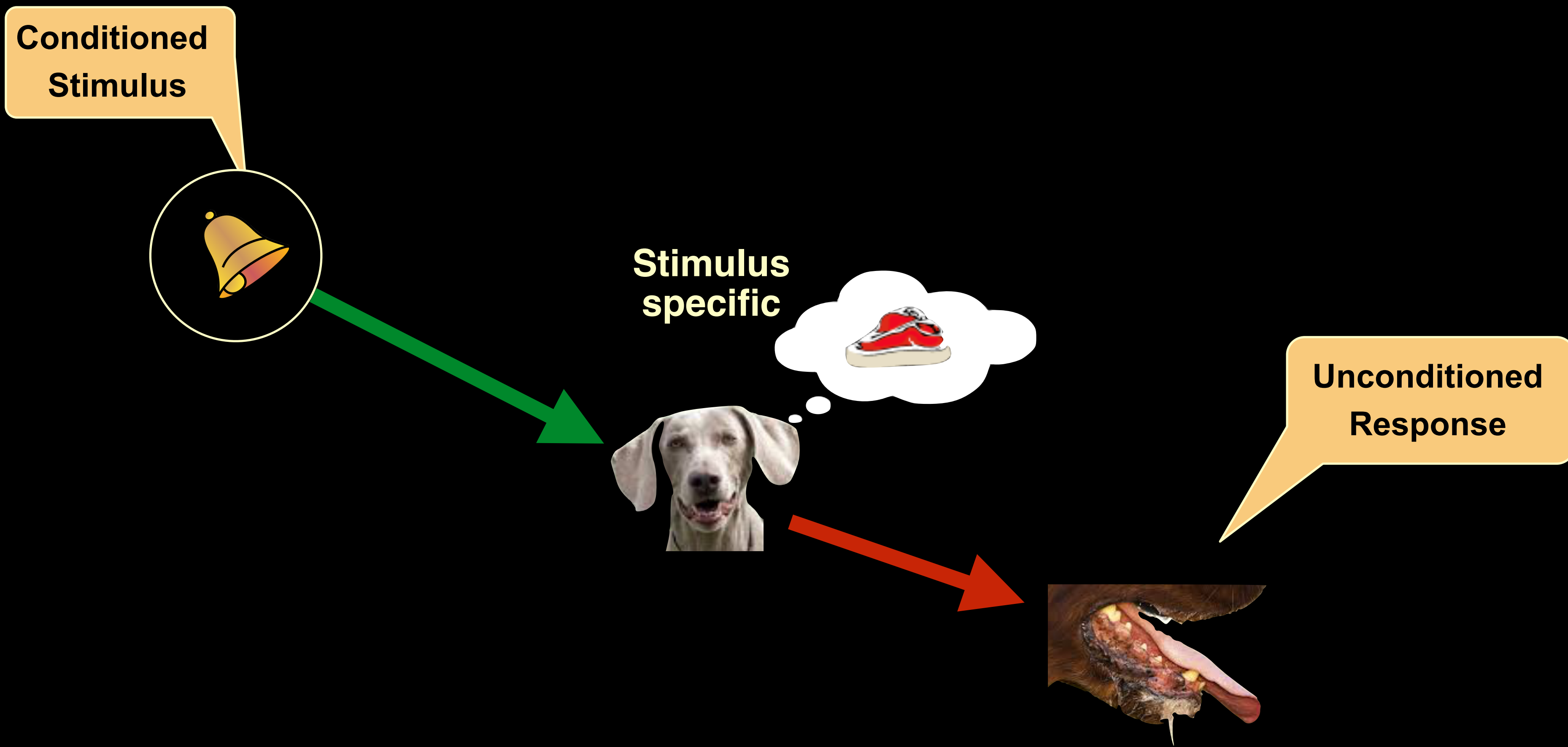
Conditioning and Prediction



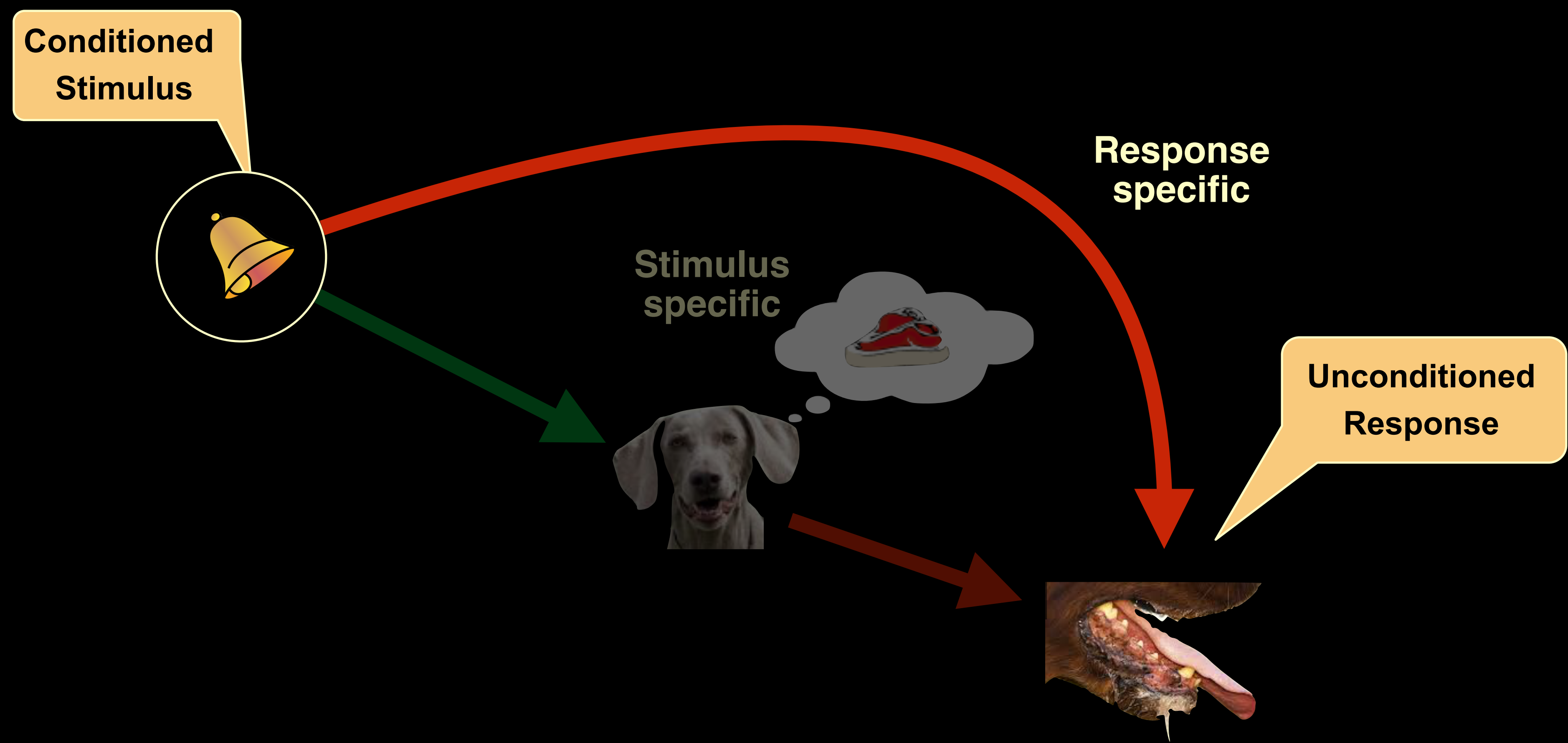
Conditioning and Prediction



Conditioning and Prediction



Conditioning and Prediction



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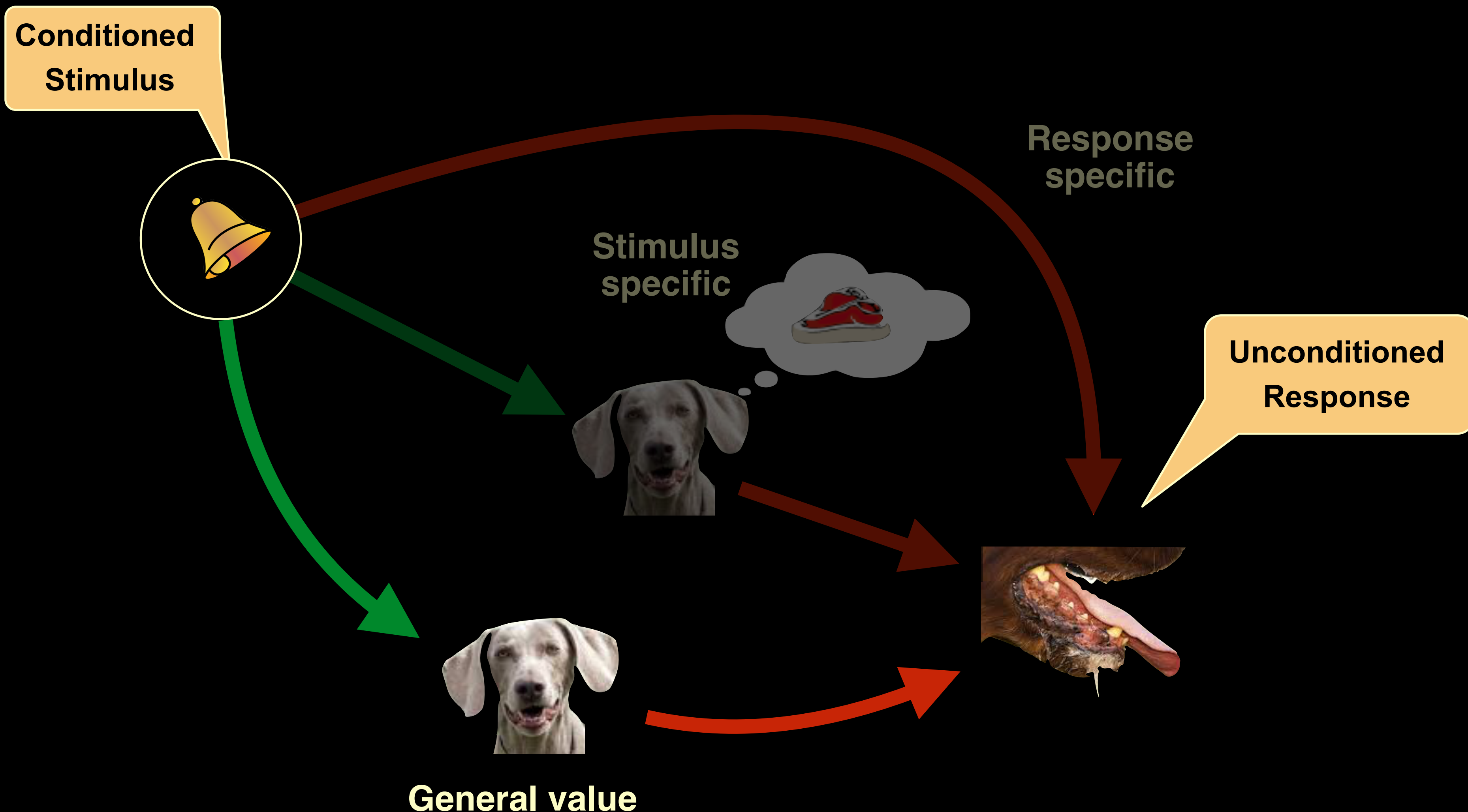


2nd Order Conditioning

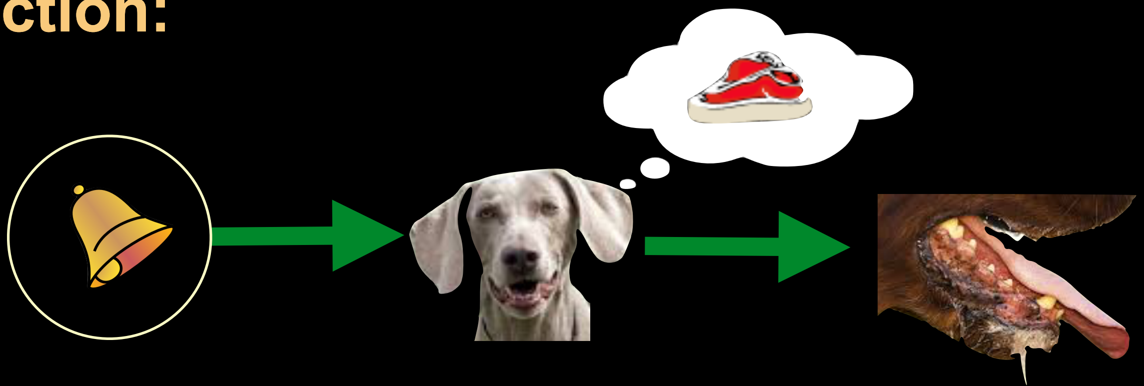
Sequential prediction:

2nd Order Conditioning

Sequential prediction:

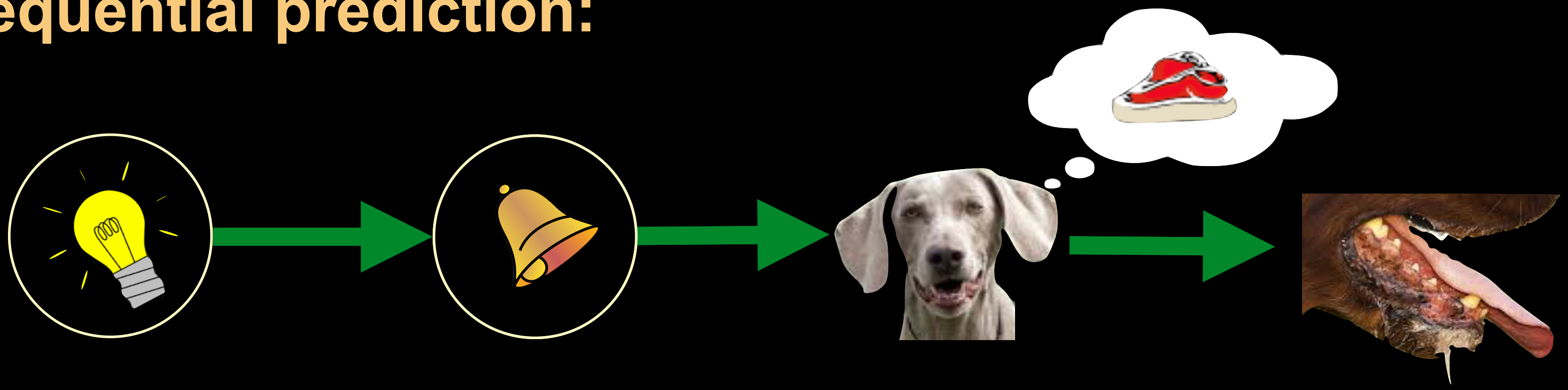
2nd Order Conditioning

Sequential prediction:



2nd Order Conditioning

Sequential prediction:



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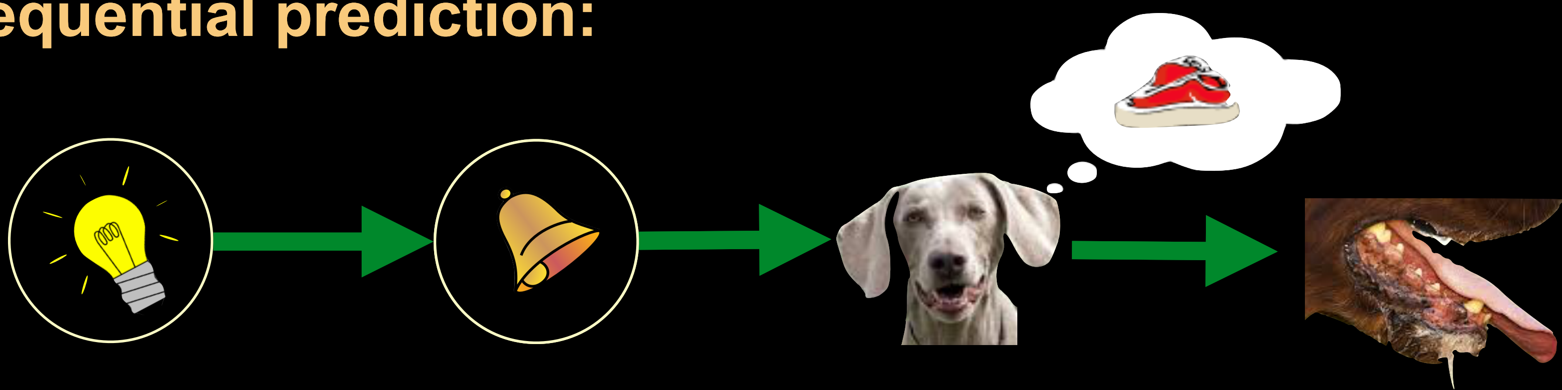

Rescorla-Wagner Rule can't handle this,

can only learn to predict the value of the current event
(cue does not generate an *actual* reward)

2nd Order Conditioning

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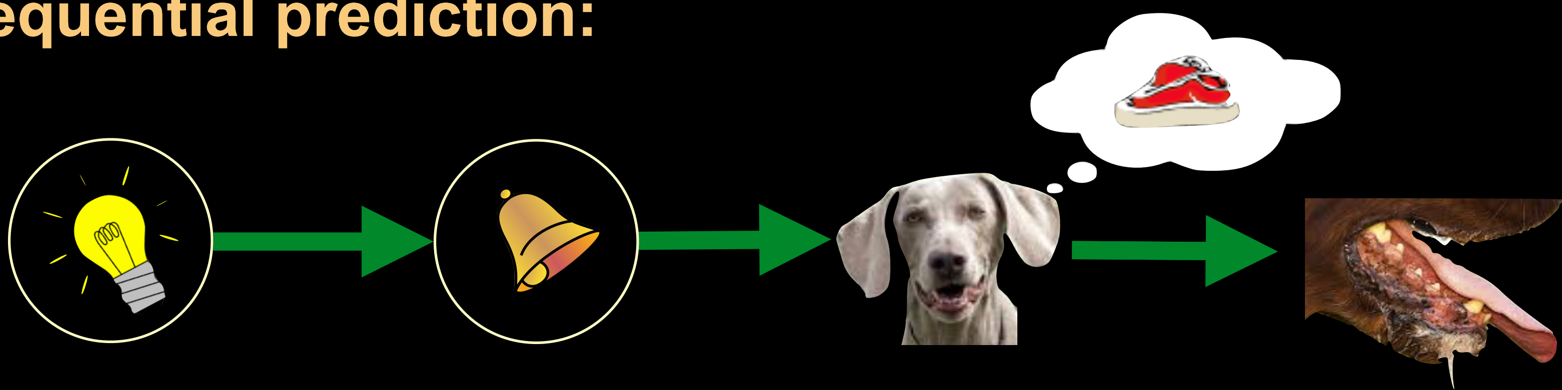

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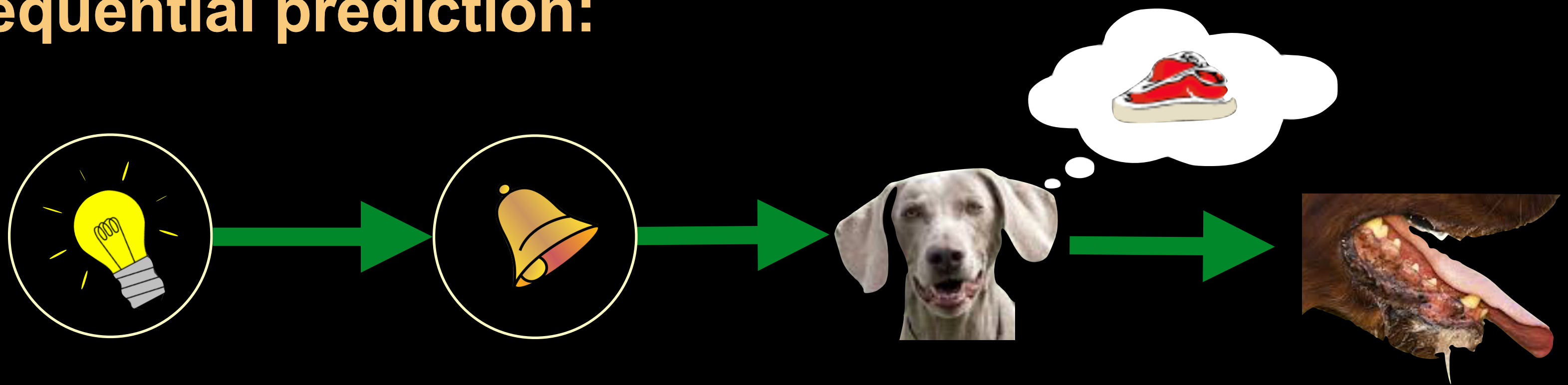

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Supervised Learning: Scalar

- Conditioning
  - Simple Prediction
    Rescorla-Wagner Rule
  - Stimulus-Action Associations
    Actor-critic model, Q Learning
- 👉 Sequence Prediction
  - Method of Temporal Differences (TD)
  - Model-Free vs. Model-Based RL
- Challenges
  - Curse of dimensionality
    - *Hierarchical RL: policies and options*
    - *State space abstraction*
  - Explore-exploit
    - ***Meta-control***

Prediction Learning

Prediction error | *Weight adjustment*

Rescorla-Wagner: $\Delta w_S = r_{S(t)} - w_{S(t)}$ $\qquad w_{S(t+1)} = w_{S(t)} + \alpha \Delta w_S$

Predict current reward:
$w_{S(t)} = r_{S(t)}$

Prediction Learning

Prediction error | *Weight adjustment*

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Currently
predicted
reward

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Predict current reward:

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Received reward

Currently predicted reward

Prediction Learning

Rescorla-Wagner:

Predict current reward:

$w_{s(t)} = r_{s(t)}$

Prediction error

$$\Delta w_s = r_{s(t)} - w_{s(t)}$$

Received reward

Currently predicted reward

Weight adjustment

$$w_{s(t+1)} = w_{s(t)} + \alpha \, \Delta w_s$$

Updated prediction

Old prediction of expected reward

Prediction Learning

Prediction error — **Weight adjustment**

Rescorla-Wagner: $\Delta w_s = r_{s(t)} - w_{s(t)}$ $\quad w_{s(t+1)} = w_{s(t)} + \alpha \, \Delta w_s$

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Received reward

Currently predicted reward

Updated prediction

Old prediction of expected reward

Temporal Difference:
(Sutton & Barto, 1981)

*Predict **all future** rewards:*

$$w_{s(t)} = r_{s(t)} + r_{s(t+1)} + r_{s(t+2)} + \ldots$$
$$= r_{s(t)} + w_{s(t+1)}$$

[Bellman equation]

by updating existing ("OLD") predictions

Prediction Learning

Prediction error | *Weight adjustment*

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Received reward

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Temporal Difference: $\Delta w_s = r_{s(t)} + w_{s(t)} - w_{s(t-1)}$

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Updated prediction

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(Sutton & Barto, 1981)

Previously expected reward

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Prediction Learning

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Received reward

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(Sutton & Barto, 1981)

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*Predict **all future** rewards:*

$$w_{s(t)} = r_{s(t)} + r_{s(t+1)} + r_{s(t+2)} + \ldots$$
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[Bellman equation]

by updating existing ("OLD") predictions

Currently received reward

Currently predicted reward

Previously expected reward

Prediction Learning

Prediction error — *Weight adjustment*

Rescorla-Wagner: $\Delta w_s = r_{s(t)} - w_{s(t)}$ $\quad w_{s(t+1)} = w_{s(t)} + \alpha \Delta w_s$

Predict current reward:

$w_{s(t)} = r_{s(t)}$

Received reward — Currently predicted reward — Updated prediction — Old prediction of expected reward

Reward Prediction Error

Temporal Difference: $\Delta w_s = r_{s(t)} + w_{s(t)} - w_{s(t-1)}$

(Sutton & Barto, 1981)

Predict ***all future*** *rewards:*

$$w_{s(t)} = r_{s(t)} + r_{s(t+1)} + r_{s(t+2)} + \ldots$$
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[Bellman equation]

by updating existing ("OLD") predictions

Currently received reward — Currently predicted reward — Previously expected reward

Prediction Learning

Prediction error | **Weight adjustment**

Rescorla-Wagner:

$$\Delta w_s = r_{s(t)} - w_{s(t)}$$

$$w_{s(t+1)} = w_{s(t)} + \alpha \Delta w_s$$

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Received reward

Currently predicted reward

Updated prediction

Old prediction of expected reward

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[Bellman equation]

by updating existing ("OLD") predictions

$$w_{s(t-1)} = w_{s(t-1)} + \alpha \Delta w_s$$

Prediction Learning

Prediction error ***Weight adjustment***

Rescorla-Wagner: $\Delta w_s = r_{s(t)} - w_{s(t)}$ $w_{s(t+1)} = w_{s(t)} + \alpha \Delta w_s$

Predict current reward:

$w_{s(t)} = r_{s(t)}$

Received reward

Currently predicted reward

Updated prediction

Old prediction of expected reward

Reward Prediction Error

Temporal Difference:
(Sutton & Barto, 1981)

*Predict **all future** rewards:*

$$w_{s(t)} = r_{s(t)} + r_{s(t+1)} + r_{s(t+2)} + \dots$$
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[Bellman equation]

by updating existing ("OLD") predictions

$$w_{s(t-1)} = w_{s(t-1)} + \alpha \Delta w_s$$

Updated "Old" prediction

"Old" reward prediction

Prediction Learning

- **Overall logic:**

Prediction Learning

- **Overall logic:**
  - ***Want:*** $V_s(t) = r_s(t)$ *(predicted value in state s at time t)*

Prediction Learning

- **Overall logic:**
  - ***Error signal:*** $\boldsymbol{\delta(t) = r_s(t) - V_s(t)}$ *(observed minus predicted)*

Prediction Learning

- **Overall logic:**
  - ***Update:*** $V_s(t+1) \leftarrow V_s(t) + \varepsilon * \delta(t)$

Prediction Learning

- **Overall logic:**
  - ***Update:*** $V_s(t+1) \leftarrow V_s(t) + \varepsilon * \delta(t)$
- **Predictions:**
  - ***Predictions*** **are** ***weights*** **that designate** ***exepcted value*****:**

Prediction Learning

- Overall logic:
  - *Update:* $V_s(t+1) \leftarrow V_s(t) + \varepsilon * \delta(t)$
- Predictions:
  - *Implicit*, not "active"

Prediction Learning

- Overall logic:
  - *Update:* $V_s(t+1) \leftarrow V_s(t) + \varepsilon * \delta(t)$
- Predictions:
  - *Conditional* (i.e., from a given state) not general

Prediction Learning

- **Overall logic:**
  - ***Update:*** $V_s(t+1) \leftarrow V_s(t) + \varepsilon * \delta(t)$
- **Predictions:**

Normally think of prediction as something active,
but easier to think about them here as weights
so you can think about existing ones you would make
in a given state

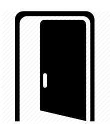
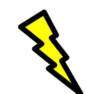
Temporal Difference Learning

Updated prediction for previous time step · *Old prediction at that time step* · *Current reward* · *Current prediction* · *Old prediction at that time step*

$$\underbrace{W_S^{new}(t-1)}_{\text{predicted value}} = W_S^{old}(t-1) + \alpha \underbrace{\left(r_S(t) + W_S^{old}(t) - W_S^{old}(t-1)\right)}_{\text{prediction error}} \qquad \alpha = 0.5$$

predicted value

prediction error

State / time step:	***1***	***2***	***3***	***4***
Stimulus / reward:	 $r(1) = 0$	 $r(2) = 0$	 $r(3) = 0$	 $r(4) = -1$

Temporal Difference Learning

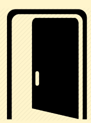
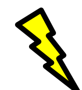
Updated prediction for previous time step | Old prediction at that time step | Current reward | Current prediction | Old prediction at that time step

$$W_S^{new}(t-1) = W_S^{old}(t-1) + \alpha \underbrace{\left(r_S(t) + W_S^{old}(t) - W_S^{old}(t-1)\right)}_{\text{prediction error}}$$

$\alpha = 0.5$

predicted value

prediction error

State / time step:	***1***	***2***	***3***	***4***
Stimulus / reward:	 $r(1) = 0$	 $r(2) = 0$	 $r(3) = 0$	 $r(4) = -1$

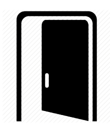
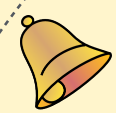
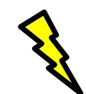
trial 1: W_S^{new}*(t-1)*

Temporal Difference Learning

Updated prediction for previous time step — *Old prediction at that time step* — *Current reward* — *Current prediction* — *Old prediction at that time step*

$$\underbrace{W_S^{new}(t-1)}_{\text{predicted value}} = W_S^{old}(t-1) + \alpha \underbrace{\left(r_S(t) + W_S^{old}(t) - W_S^{old}(t-1)\right)}_{\text{prediction error}}$$

α = 0.5

State / time step:	***1***	***2***	***3***	***4***
Stimulus / reward:	 $r(1) = 0$	 $r(2) = 0$	 $r(3) = 0$	 $r(4) = -1$

trial 1*:* $W_S^{new}(t\text{-}1)$ $\quad 0 + 0.5\underbrace{(0 + 0 - 0)}_{0}$ $\quad$ ***= 0***

Temporal Difference Learning

Updated prediction for previous time step — *Old prediction at that time step* — *Current reward* — *Current prediction* — *Old prediction at that time step*

$$\underbrace{W_S^{new}(t-1)}_{\text{predicted value}} = W_S^{old}(t-1) + \alpha \underbrace{\left(r_S(t) + W_S^{old}(t) - W_S^{old}(t-1)\right)}_{\text{prediction error}}$$

α = 0.5

State / time step:	***1***	***2***	***3***	***4***
Stimulus / reward:	$r(1) = 0$	$r(2) = 0$	$r(3) = 0$	$r(4) = -1$
trial 1: W_S^{new}*(t-1)*	$0 + 0.5\underbrace{(0+0-0)}_{0}$ ***= 0***	$0 + 0.5\underbrace{(0+0-0)}_{0}$ ***= 0***		

Temporal Difference Learning

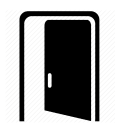
Updated prediction for previous time step | *Old prediction at that time step* | *Current reward* | *Current prediction* | *Old prediction at that time step*

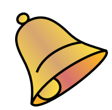
$$\underbrace{W_S^{new}(t-1)}_{\text{predicted value}} = W_S^{old}(t-1) + \alpha\,\underbrace{\left(r_S(t) + W_S^{old}(t) - W_S^{old}(t-1)\right)}_{\text{prediction error}}$$

$\alpha = 0.5$

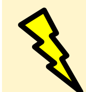
State / time step: **1** **2** **3** **4**

Stimulus / reward:

 $r(1) = 0$

 $r(2) = 0$

 $r(3) = 0$

 $r(4) = -1$

trial 1: $W_S^{new}(t\text{-}1)$

$0 + 0.5\underbrace{(0+0-0)}_{0}$ $= 0$

$0 + 0.5\underbrace{(0+0-0)}_{0}$ $= 0$

$0 + 0.5\underbrace{(-1+0-0)}_{-1}$ $= -0.5$

Temporal Difference Learning

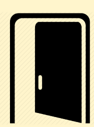
Updated prediction for previous time step — *Old prediction at that time step* — *Current reward* — *Current prediction* — *Old prediction at that time step*

$$\underbrace{W_S^{new}(t-1)}_{\text{predicted value}} = W_S^{old}(t-1) + \alpha\ \underbrace{\left(r_S(t) + W_S^{old}(t) - W_S^{old}(t-1)\right)}_{\text{prediction error}}$$

α = 0.5

State / time step: ***1*** ***2*** ***3*** ***4***

Stimulus / reward:

 $r(1) = 0$

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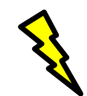
 $r(4) = -1$

$0 + 0.5\underbrace{(0 + 0 - 0)}_{0}$ *= 0*

$0 + 0.5\underbrace{(0 + 0 - 0)}_{0}$ *= 0*

$0 + 0.5\underbrace{(-1 + 0 - 0)}_{-1}$ *= -0.5*

trial 2: W_S^{new}*(t-1)*

Temporal Difference Learning

Updated prediction for previous time step — *Old prediction at that time step* — *Current reward* — *Current prediction* — *Old prediction at that time step*

$$\underbrace{W_S^{new}(t-1)}_{\text{predicted value}} = W_S^{old}(t-1) + \alpha \underbrace{\left(r_S(t) + W_S^{old}(t) - W_S^{old}(t-1)\right)}_{\text{prediction error}}$$

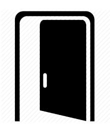
$\alpha = 0.5$

predicted value

prediction error

State / time step: **1** **2** **3** **4**

Stimulus / reward:

 $r(1) = 0$

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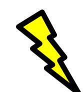
 $r(4) = -1$

$0 + 0.5\underbrace{(0+0-0)}_{0}$ = 0

$0 + 0.5\underbrace{(0+0-0)}_{0}$ = 0

$0 + 0.5\underbrace{(-1+0-0)}_{-1}$ = -0.5

trial 2: $W_S^{new}(t\text{-}1)$ $0 + 0.5\underbrace{(0+0-0)}_{0}$ **= 0**

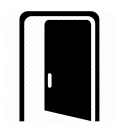
Temporal Difference Learning

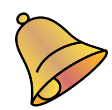
Updated prediction for previous time step · *Old prediction at that time step* · *Current reward* · *Current prediction* · *Old prediction at that time step*

$$\underbrace{W_S^{new}(t-1)}_{\text{predicted value}} = W_S^{old}(t-1) + \alpha \underbrace{\left(r_S(t) + W_S^{old}(t) - W_S^{old}(t-1)\right)}_{\text{prediction error}} \qquad \alpha = 0.5$$

State / time step: **1** **2** **3** **4**

Stimulus / reward:

 $r(1) = 0$

 $r(2) = 0$

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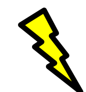
 $r(4) = -1$

Time step 1: $0 + 0.5\underbrace{(0 + 0 - 0)}_{0}$ $= 0$

Time step 2: $0 + 0.5\underbrace{(0 + 0 - 0)}_{0}$ $= 0$

Time step 3: $0 + 0.5\underbrace{(-1 + 0 - 0)}_{-1}$ $= -0.5$

trial 2: $W_S^{new}(t\text{-}1)$

Time step 1: $0 + 0.5\underbrace{(0 + 0 - 0)}_{0}$ $= 0$

Time step 2: $0 + 0.5\underbrace{(0 - 0.5 - 0)}_{-0.5}$ $= -0.25$

Temporal Difference Learning

Updated prediction for previous time step — *Old prediction at that time step* — *Current reward* — *Current prediction* — *Old prediction at that time step*

$$\underbrace{W_S^{new}(t-1)}_{\text{predicted value}} = W_S^{old}(t-1) + \alpha\ \underbrace{\left(r_S(t) + W_S^{old}(t) - W_S^{old}(t-1)\right)}_{\text{prediction error}}$$

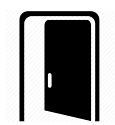
α = 0.5

predicted value

prediction error

State / time step: ***1*** ***2*** ***3*** ***4***

Stimulus / reward: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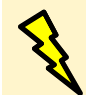
$r(4) = -1$

$0 + 0.5\underbrace{(0 + 0 - 0)}_{0}$ *= 0*

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***trial 2**:* W_S^{new}*(t-1)*

$0 + 0.5\underbrace{(0 + 0 - 0)}_{0}$ *= 0*

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$-0.5 + 0.5\underbrace{(-1 + 0 + 0.5)}_{-0.5}$ ***= -0.75***

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$$\underbrace{W_S^{new}(t-1)}_{\text{predicted value}} = W_S^{old}(t-1) + \alpha \underbrace{\left(r_S(t) + W_S^{old}(t) - W_S^{old}(t-1)\right)}_{\text{prediction error}}$$

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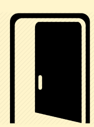
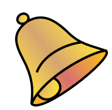
 $r(1) = 0$

 $r(2) = 0$

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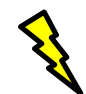 $r(4) = -1$

1	2	3
$0 + 0.5\underbrace{(0+0-0)}_{0}$ *= 0*	$0 + 0.5\underbrace{(0+0-0)}_{0}$ *= 0*	$0 + 0.5\underbrace{(-1+0-0)}_{-1}$ *= -0.5*
$0 + 0.5\underbrace{(0+0-0)}_{0}$ *= 0*	$0 + 0.5\underbrace{(0-0.5-0)}_{-0.5}$ *= -0.25*	$-0.5 + 0.5\underbrace{(-1+0+0.5)}_{-0.5}$ *= -0.75*

trial 3: W_S^{new}*(t-1)*

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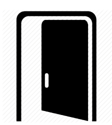
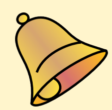
 $r(1) = 0$

 $r(2) = 0$

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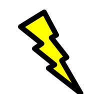 $r(4) = -1$

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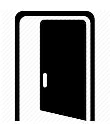
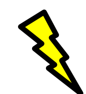
trial 3: $W_S^{new}(t\text{-}1)$ $0 + 0.5\underbrace{(0 - 0.25 - 0)}_{-0.25}$ $= -0.125$

Temporal Difference Learning

Updated prediction for previous time step | Old prediction at that time step | Current reward | Current prediction | Old prediction at that time step

$$\underbrace{W_S^{new}(t-1)}_{\text{predicted value}} = W_S^{old}(t-1) + \alpha\ \underbrace{\left(r_S(t) + W_S^{old}(t) - W_S^{old}(t-1)\right)}_{\text{prediction error}}$$

$\alpha = 0.5$

State / time step:	1	2	3	4
Stimulus / reward:	 $r(1) = 0$	 $r(2) = 0$	 $r(3) = 0$	 $r(4) = -1$
	$0 + 0.5\underbrace{(0 + 0 - 0)}_{0}$ $= 0$	$0 + 0.5\underbrace{(0 + 0 - 0)}_{0}$ $= 0$	$0 + 0.5\underbrace{(-1 + 0 - 0)}_{-1}$ $= -0.5$	
	$0 + 0.5\underbrace{(0 + 0 - 0)}_{0}$ $= 0$	$0 + 0.5\underbrace{(0 - 0.5 - 0)}_{-0.5}$ $= -0.25$	$-0.5 + 0.5\underbrace{(-1 + 0 + 0.5)}_{-0.5}$ $= -0.75$	
trial 3: $W_S^{new}(t\text{-}1)$	$0 + 0.5\underbrace{(0 - 0.25 - 0)}_{-0.25}$ $= -0.125$	$-0.25 + 0.5\underbrace{(0 - 0.75 + 0.25)}_{-0.5}$ $= -0.5$		

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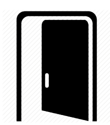
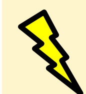
State / time step:	*1*	*2*	*3*	*4*
Stimulus / reward:	 $r(1) = 0$	 $r(2) = 0$	 $r(3) = 0$	 $r(4) = -1$
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Schulz, 1993; Montague, Dayan & Sejnowski, 1996

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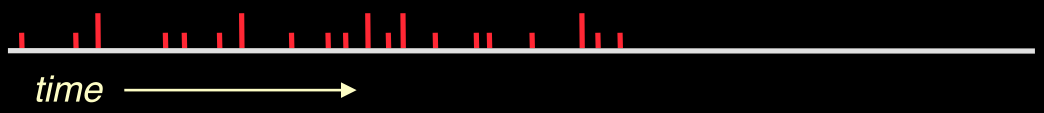
Before learning



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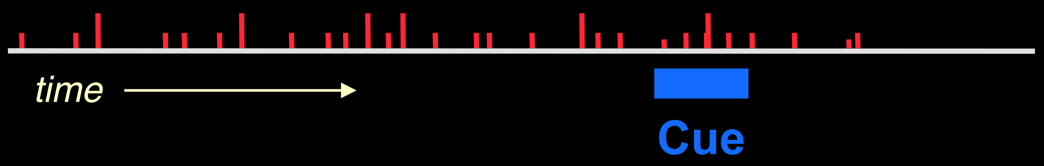

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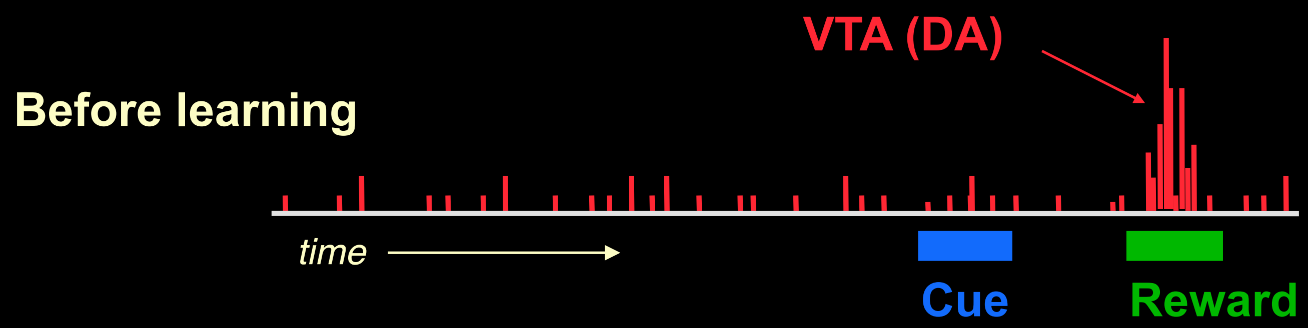

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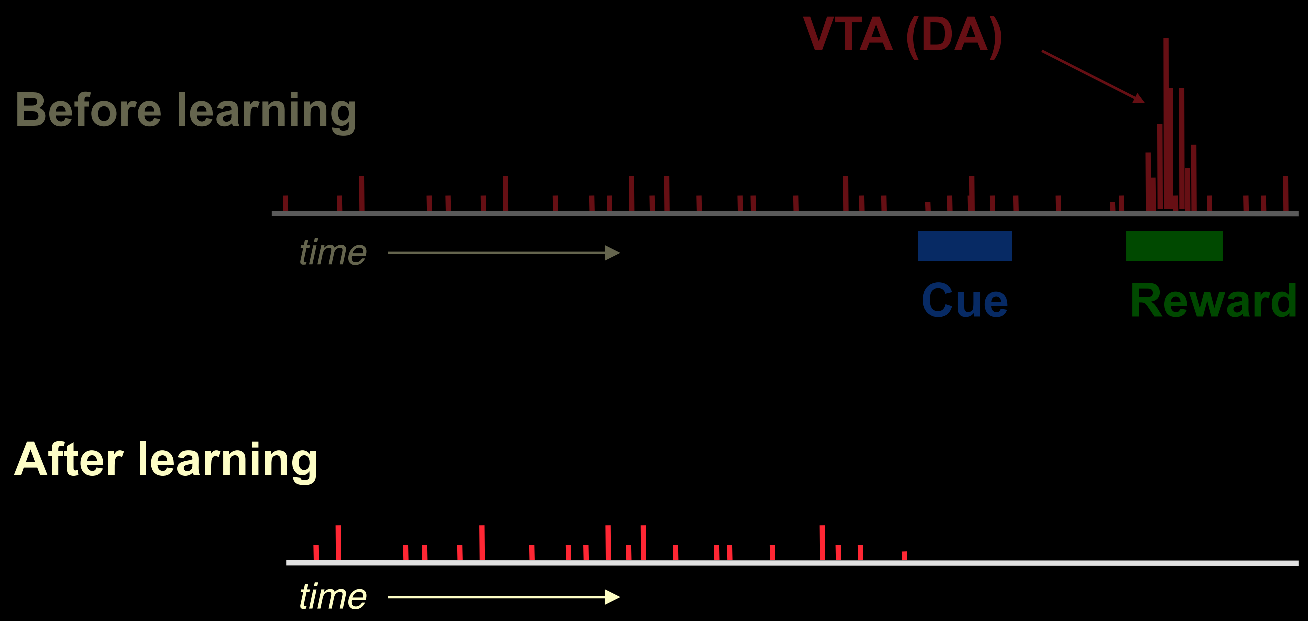

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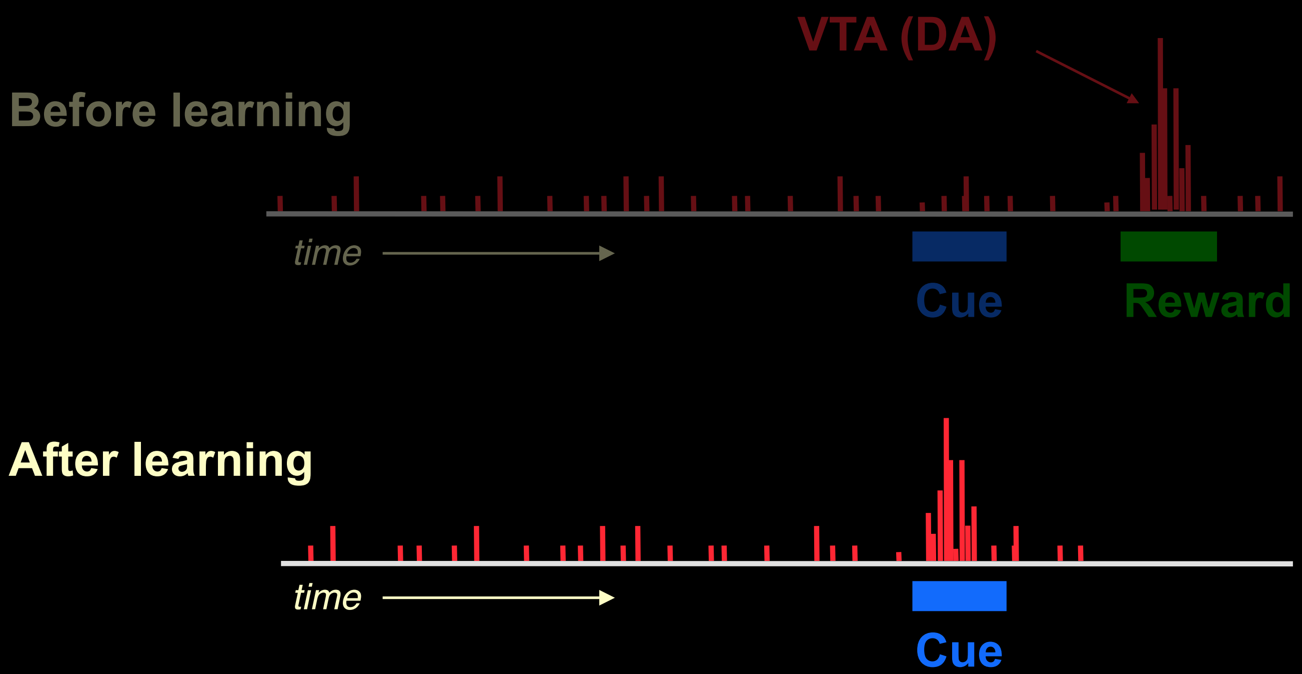

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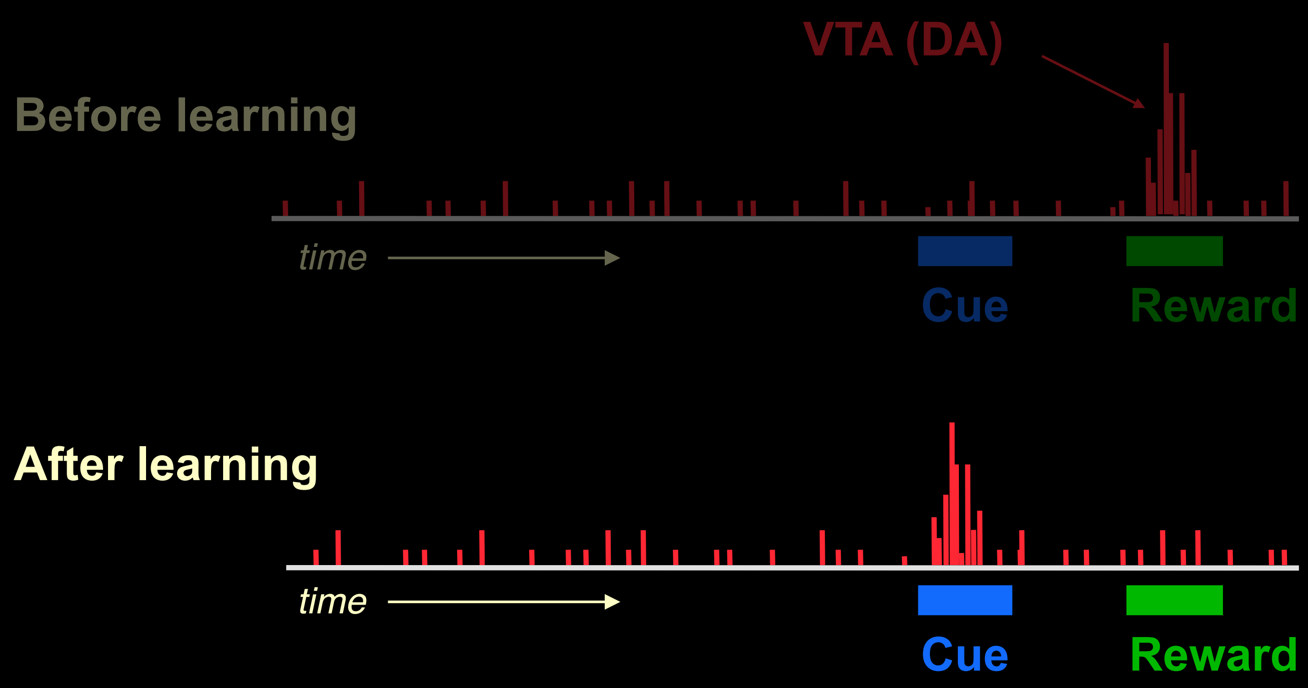

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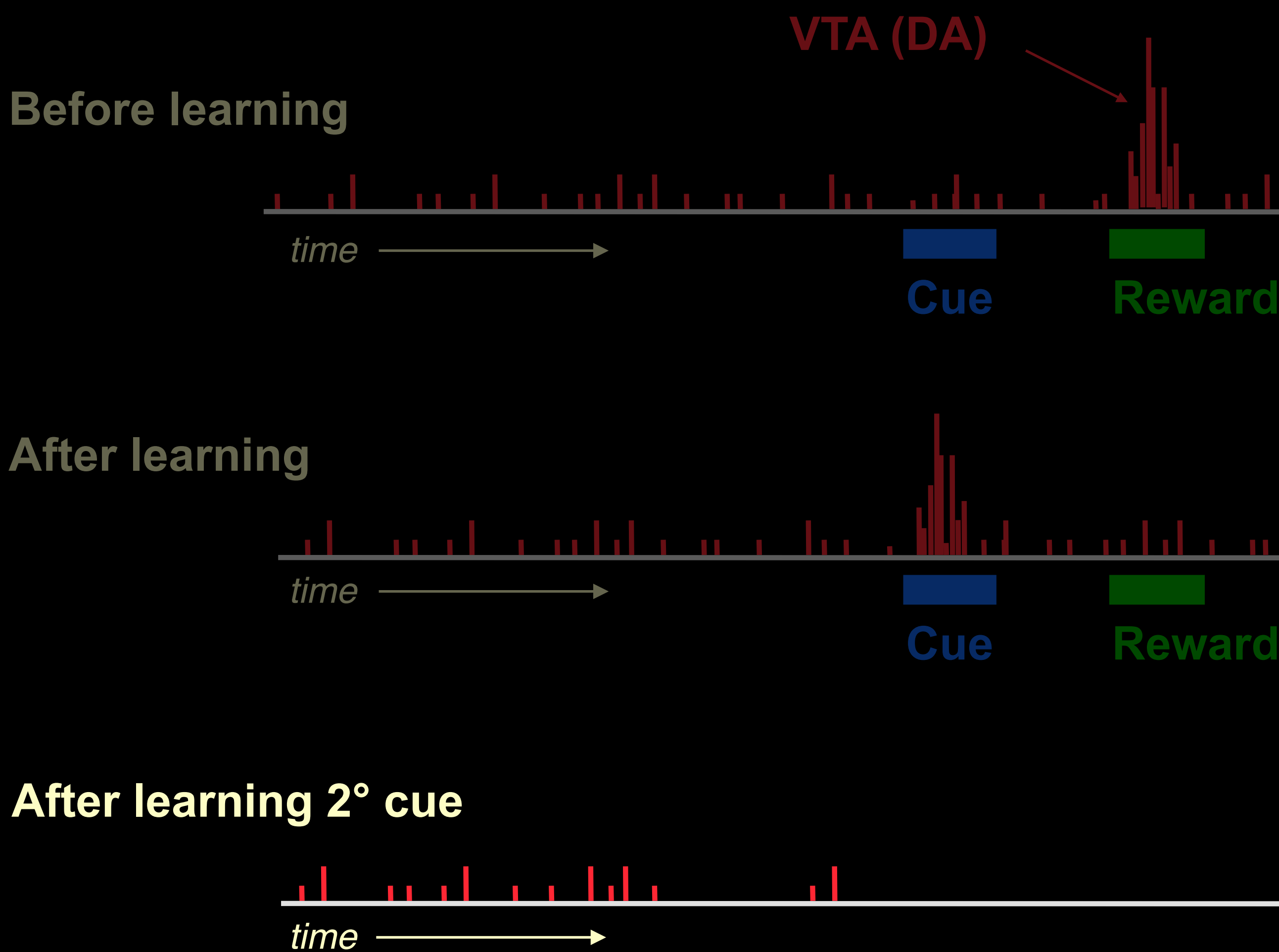

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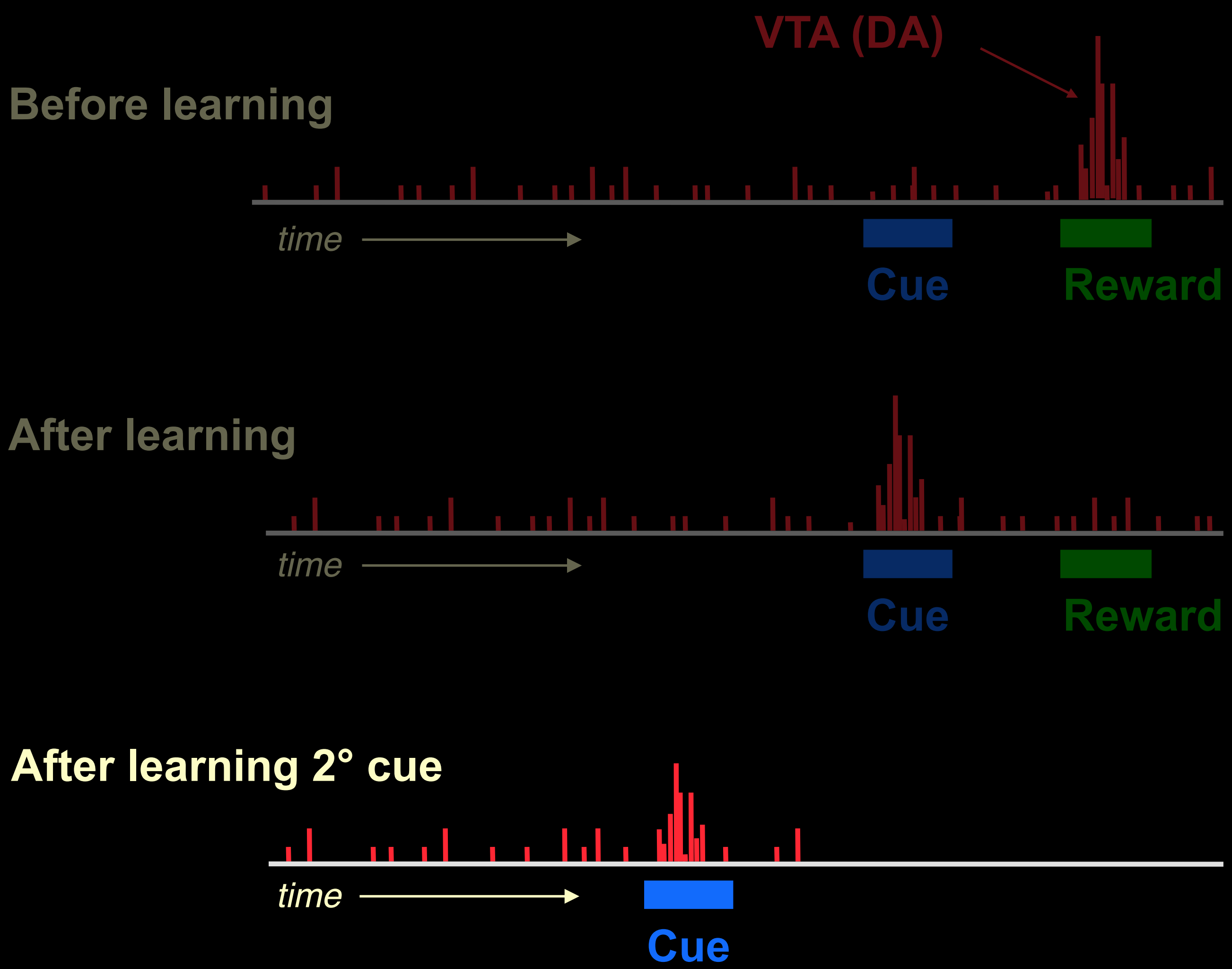

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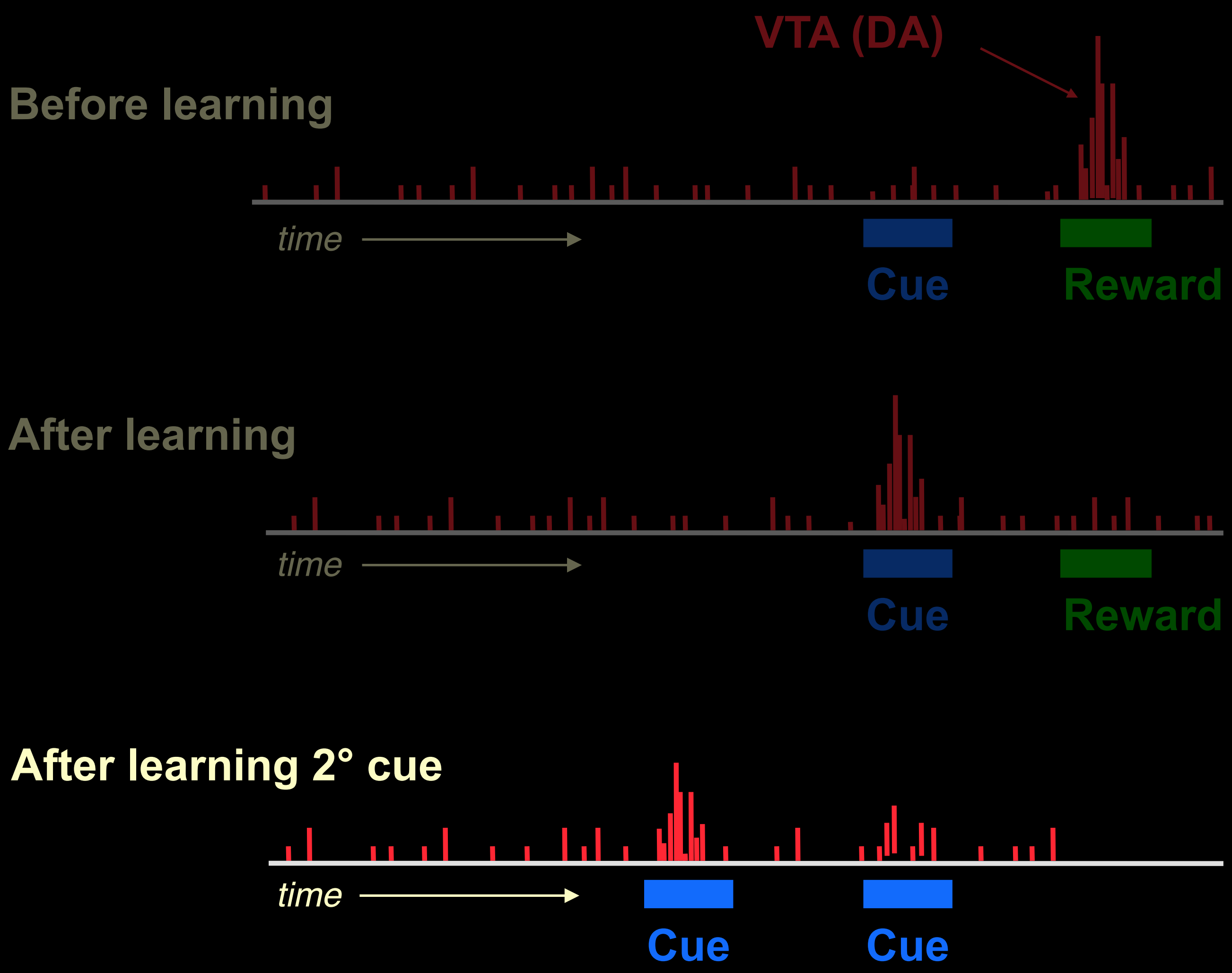

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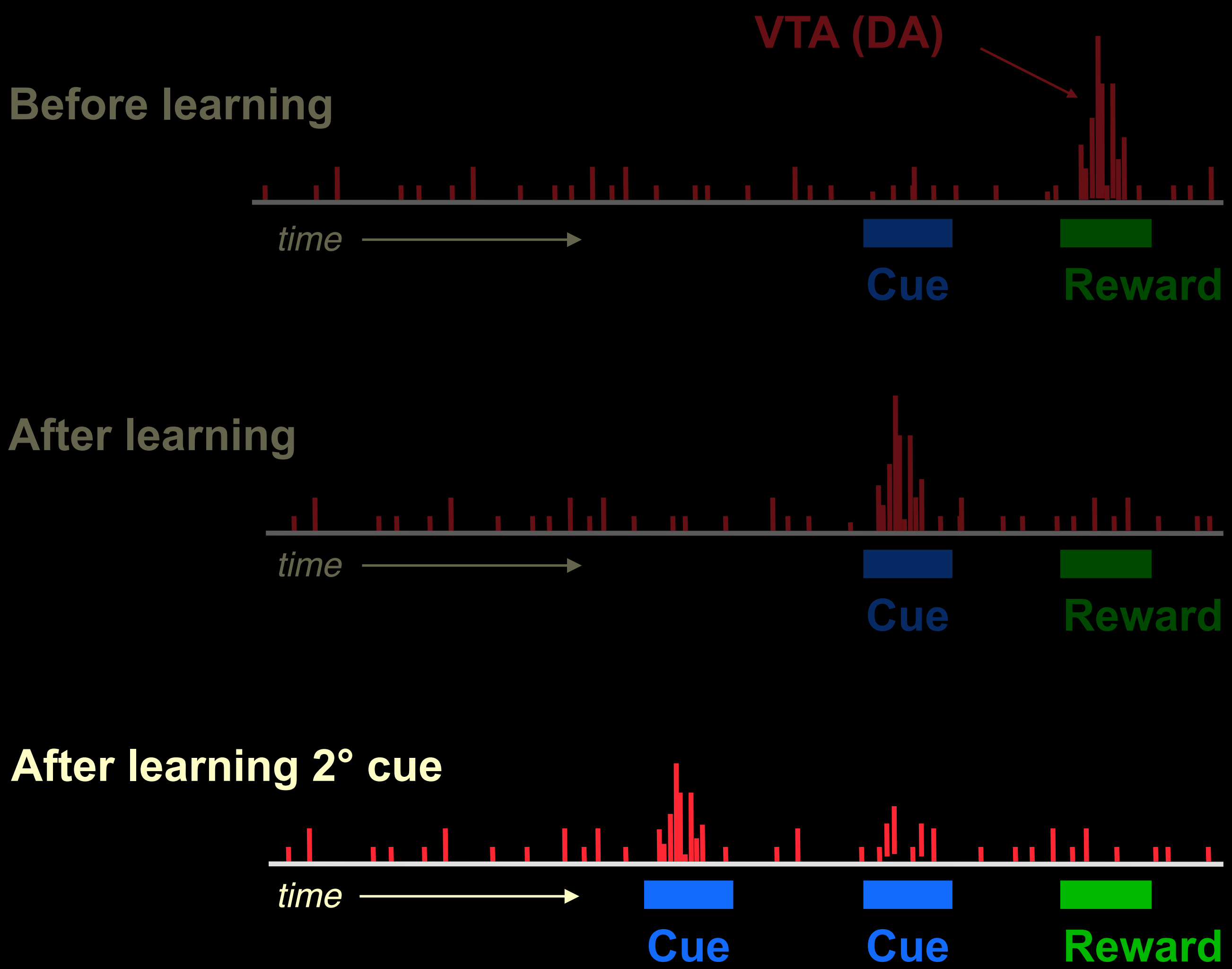

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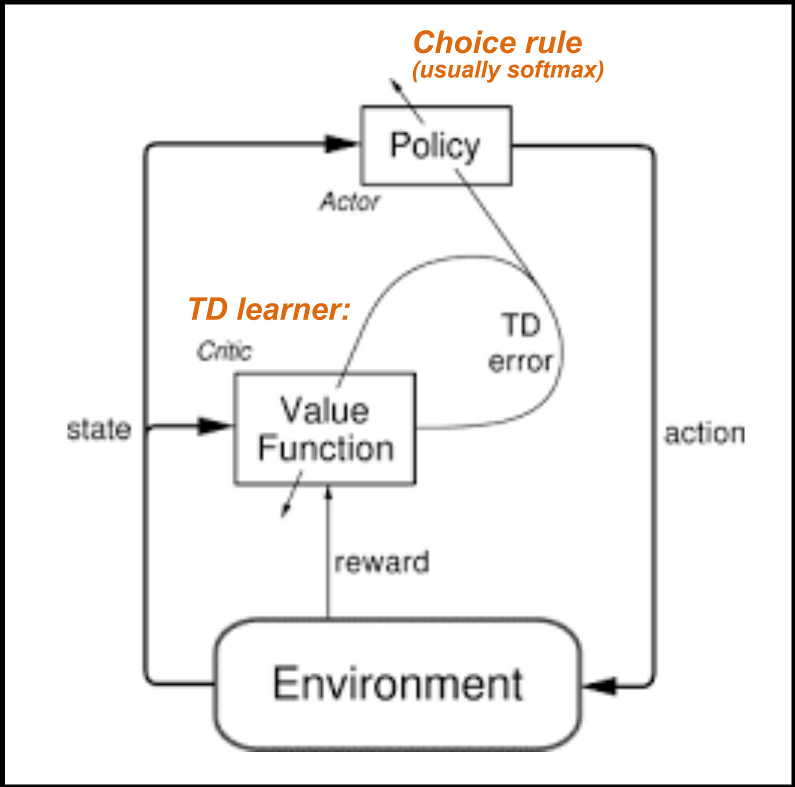

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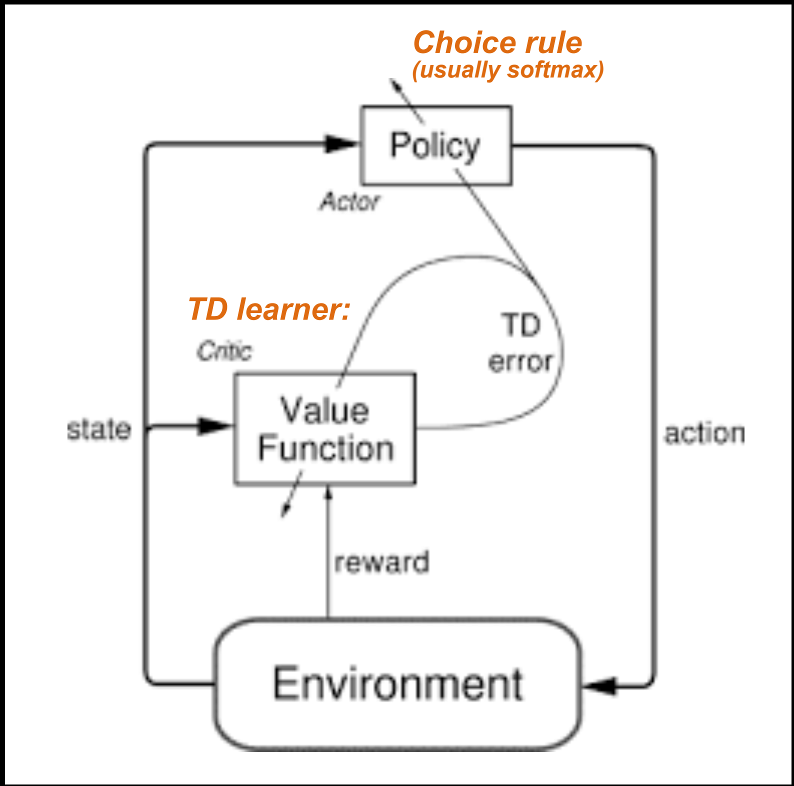

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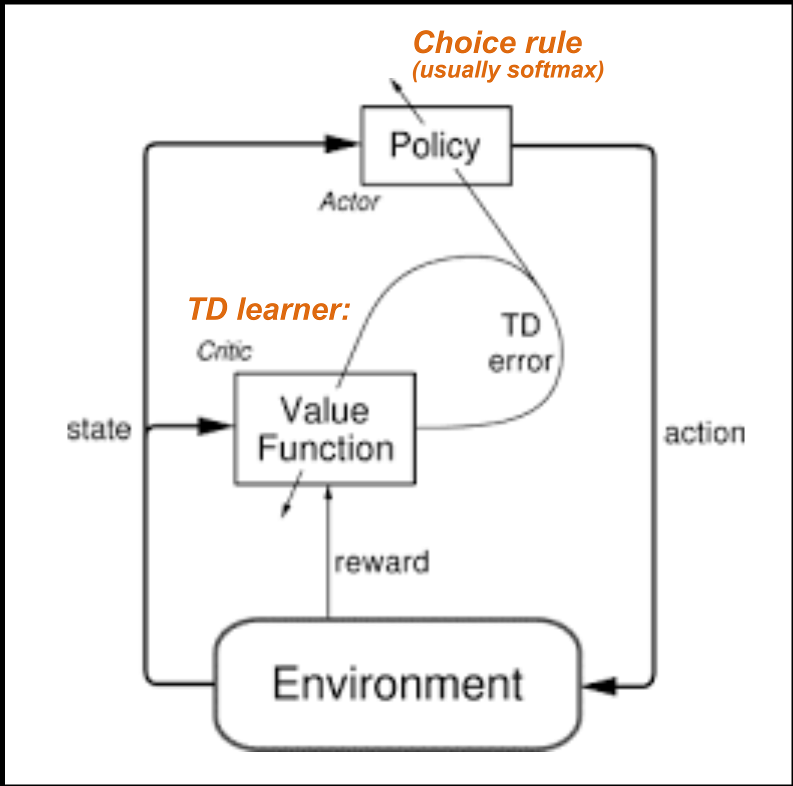

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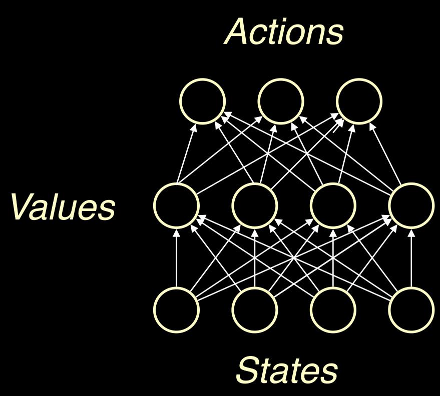

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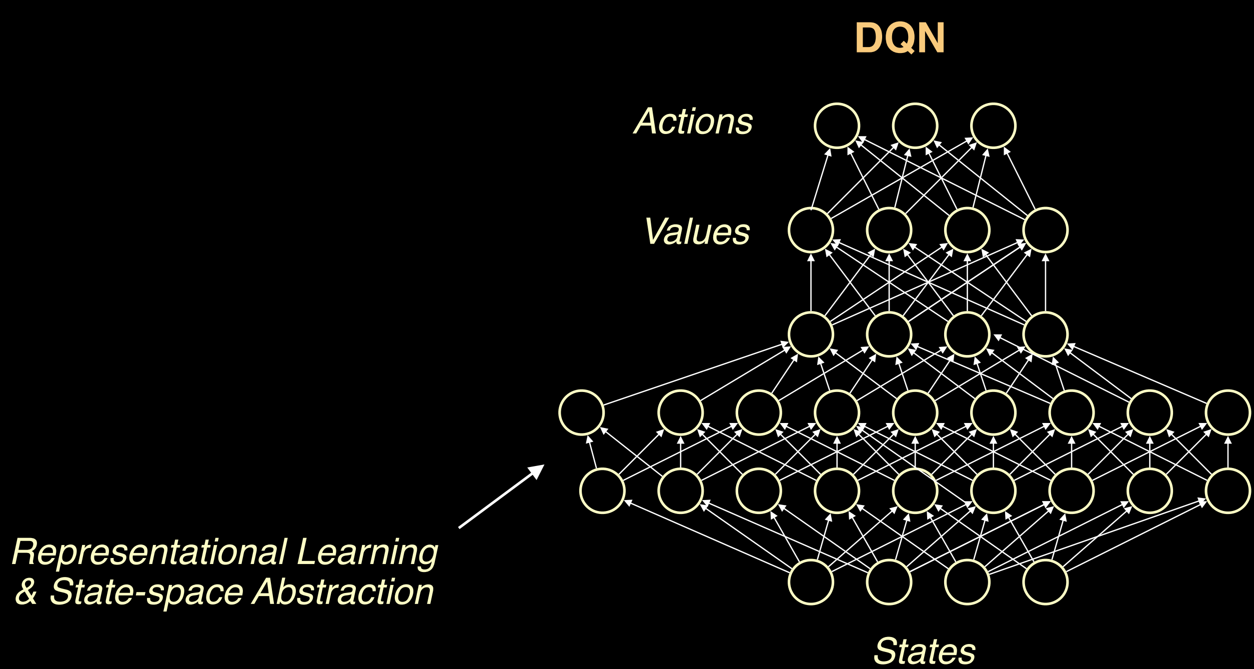

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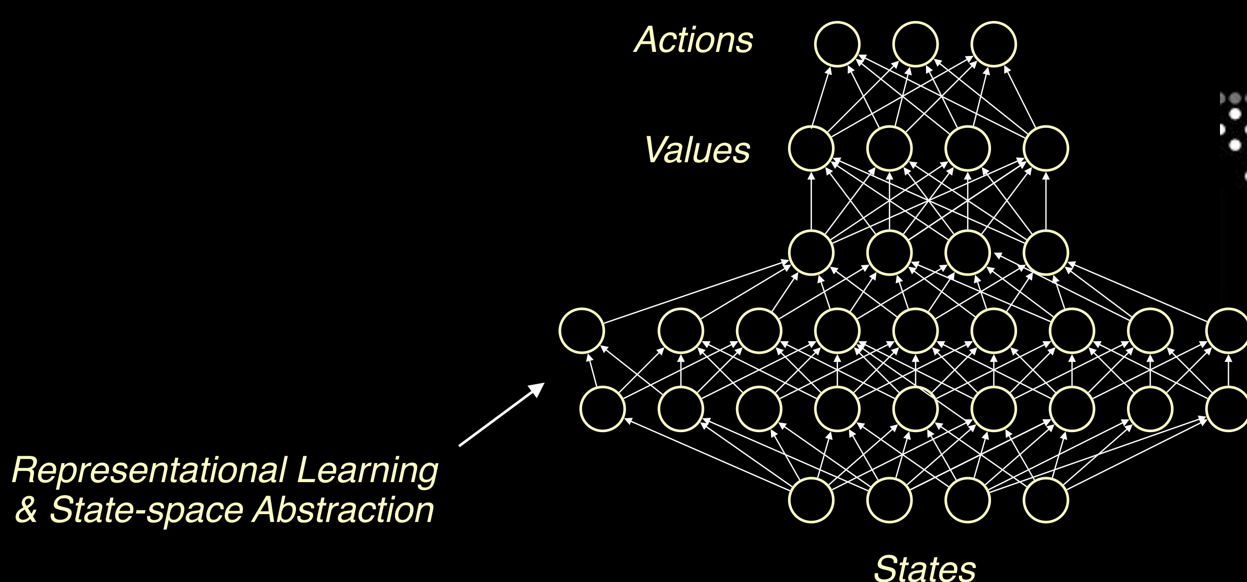

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  - **Problem:**
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Supervised Learning: Scalar

- Conditioning
  - Simple Prediction
    - *Rescorla-Wagner Rule*
  - Stimulus-Action Associations
    - *Actor-critic model, Q Learning*
- Sequence Prediction
  - Method of Temporal Differences (TD)
  - Model-Free vs. Model-Based RL
- Challenges
  - Curse of dimensionality
    - *State space abstraction*
    - *Hierarchical RL: policies and options*
  - Explore-exploit
    - ***Meta-control***